Integrated Math 3
Unit 1: Analytic Geometry
1.13

Name: _____

Date: _____ Period: _____

Objective: Complete the square to find the center and radius of a circle given by an equation.

<u>**Warm Up:**</u>
Factor:

a. $x^2 - 16x + 15$

$$(x-15)(x-1)$$

b. $x^2 - 14x + 49$

$$(x-7)(x-7)$$

<u>**Steps for Completing the Square:**</u>

1. Be sure that the coefficient of the highest power is <u>one</u>. If it's not, <u>factor</u> each term by that value to create a leading coefficient of <u>one</u>.
2. Move the constant term to the right hand side.
3. Prepare to add the needed value to create the perfect square trinomial. Be sure to <u>balance</u> the equation.
4. To find the needed value for the perfect square trinomial, take <u>half</u> of the coefficient of the <u>middle</u> term, <u>square</u> it, and add that value to both sides of the equation.
5. <u>Factor</u> the perfect square trinomial.

<u>**Examples:**</u> Convert the general form circle equations to standard form. Label the center and radius.

a. $x^2 + y^2 - 8x + 6y - 24 = 0$

$x^2 (-8x) + \underline{16} + y^2 (6y) + \underline{9} = 24 + \underline{16} + \underline{9}$

$(-\frac{8}{2})^2 = 16 \qquad (\frac{6}{2})^2 = 9$

$(x - 8x + 16) + (y^2 + 6y + 9) = 49$

$(x-4)^2 + (y+3)^2 = 49$

center: $(4, -3)$

radius: 7

b. $6x^2 - 12x + 6y^2 + 36y = 36$

$6(x^2 (-2x) + \underline{1}) + 6(y^2 (6y) + \underline{9})$

$= 36 + \underline{6} + \underline{54}$

$(\frac{-2}{2})^2 = 1 \qquad\qquad (\frac{6}{2})^2 = 9$

$\dfrac{6(x-1)^2 + 6(y+3)^2}{6} = \dfrac{96}{6}$

$(x-1)^2 + (y+3)^2 = 16$

center: $(1, -3)$

radius: 4

Examples: Convert the general form circle equations to standard form. Label the center and radius.

a. $24x + x^2 + 6y + y^2 + 137 = 0$

$\left(x^2 + \boxed{24}x + \underline{144}\right) + \left(y^2 + 6y + \underline{9}\right) = -137 + \underline{144} + \underline{9}$

$\left(\frac{24}{2}\right)^2 = 144 \qquad \left(\frac{6}{2}\right)^2 = 9$

$(x+12)^2 + (y+3)^2 = 16$

center: $(-12, -3)$

radius: 4

b. $x^2 + y^2 - 8x + 6y + 25 = 0$

$\left(x^2 - 8x + \underline{16}\right) + \left(y^2 + 6y + \underline{9}\right) = -25 + \underline{16} + \underline{9}$

$\left(\frac{-8}{2}\right)^2 = 16 \qquad \left(\frac{6}{2}\right)^2 = 9$

$(x-4)^2 + (y+3)^2 = 0$

center: $(4, -3)$

radius: 0

$*$ forms just a point! $*$

c. $8x + 32y + y^2 = -263 - x^2$

$\left(x^2 + 8x + \underline{16}\right) + \left(y^2 + 32y + \underline{256}\right) = -263 + \underline{16} + \underline{256}$

$\left(\frac{8}{2}\right)^2 = 16 \qquad \left(\frac{32}{2}\right)^2 = 256$

$(x+4)^2 + (y+16)^2 = 9$

Center: $(-4, -16)$

radius: 3

d. $364 + 28y + y^2 + x^2 = -26x$

$\left(x^2 + 26x + \underline{169}\right) + \left(y^2 + 28y + \underline{196}\right) = -364 + \underline{169} + \underline{196}$

$\left(\frac{26}{2}\right)^2 = 169 \qquad \left(\frac{28}{2}\right)^2 = 196$

$(x+13)^2 + (y+14)^2 = 1$

Center: $(-13, -14)$

radius: 1