Integrated Math 3
Unit 2: Geometric Proofs & Modeling
2.2

Name: ____________________

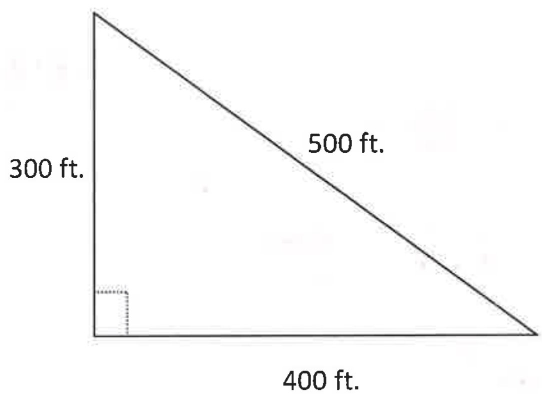
Date: __________ Period: ______

Objective: to apply volume and surface area to density

Warm up: How can you calculate density?

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{m}{V}$$

Example 1: Farmer Brown has a large pasture but only a limited amount of fencing. He knows that each cow needs 3000 square feet of pasture to grow and produce in a healthy, sustainable, manner. Currently, his fence is used to make a pen that is a right triangle as shown below.



a. How many cows can this current pen sustain?

$$A = \tfrac{1}{2}bh = \tfrac{1}{2}(400)(300) = \frac{60{,}000 \text{ ft}^2}{3{,}000 \text{ ft}^2}$$

$\boxed{20 \text{ cows}}$

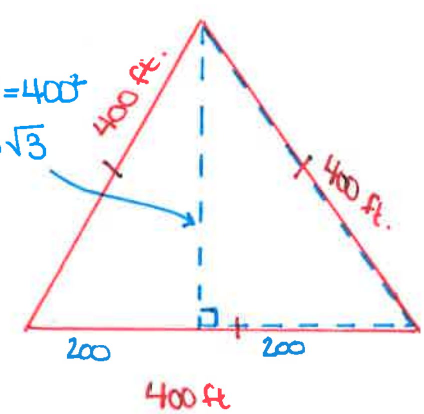
b. If Farmer Brown decides to use the same amount of fence to build a pen that is an equilateral triangle, how many cows could he then sustain?

Perimeter: $300 + 400 + 500 = 1200$ total feet of fencing

$\frac{1200 \text{ ft}}{3} = 400$ ft.

$200^2 + b^2 = 400^2$

$b = 200\sqrt{3}$

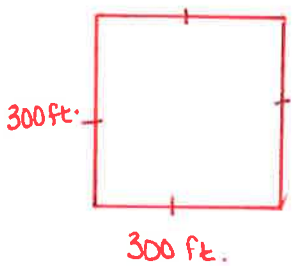


$$A = \tfrac{1}{2}bh = \tfrac{1}{2}(400)(200\sqrt{3}) \approx \frac{69{,}282 \text{ ft}^2}{3{,}000 \text{ ft}^2}$$

$\boxed{23 \text{ cows}}$

c. Suppose Farmer Brown decides to use the same amount of fence to build a rectangular pen. Design the rectangular pen that would maximize the pasture area. What would be the dimensions and area of this new pen? How many cows could it sustain?

Perimeter = 1200 ft.

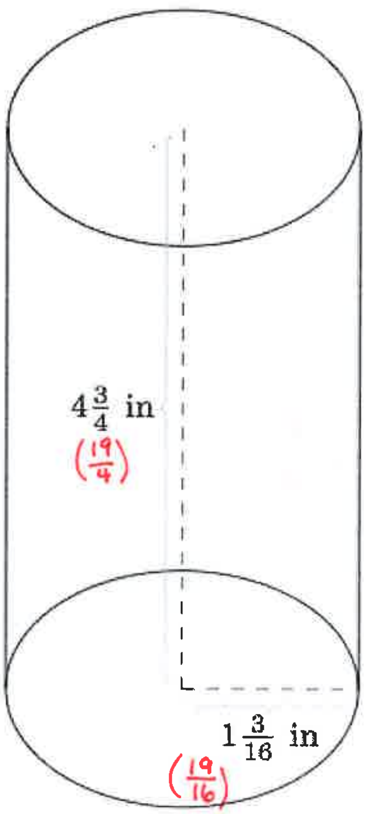


$A = bh = 300(300) = \dfrac{90{,}000 \text{ ft}^2}{3{,}000 \text{ ft}^2}$ $\boxed{30 \text{ cows}}$

Example 2: A cylindrical soda can is made of aluminum. It is approximately $4\frac{3}{4}$ inches high and the top and bottom have a radius of approximately $1\frac{3}{16}$ inches:

$\frac{19}{4}$ in.

$\frac{19}{16}$ in.



a. Find the approximate surface area of the soda can.

top: $\pi r^2 = \pi\left(\frac{19}{16}\right)^2 = \frac{361}{256}\pi$

bottom: $\frac{361}{256}\pi$

Sides: $(2\pi r)h = 2\pi\left(\frac{19}{16}\right)\cdot\frac{19}{4} = \frac{361}{32}\pi$

$SA = \frac{361}{256}\pi + \frac{361}{256}\pi + \frac{361}{32}\pi = \boxed{\frac{1805}{128}\pi \text{ in}^2}$

b. The density of aluminum is approximately 2.70 grams per cubic centimeter. If the mass of the soda can is approximately 15 grams, how many cubic centimeters of aluminum does it contain?

$d = \frac{m}{v}$

$\frac{2.70 \text{ g/cm}^3}{1} = \frac{15 \text{ g}}{v}$

$\frac{2.70v}{2.70} = \frac{15}{2.70}$

$\boxed{v \approx 5.56 \text{ cm}^3}$

Example 3: The population of Lindenhurst as of 2010 is 14,462. This has been a steadily increasing for the last 20 years. The borders of Lindenhurst have also been expanding. Currently Lindenhurst covers 4.78 miles^2.

a. Find the density of Lindenhurst (be sure to include the correct label).

$$d = \frac{14{,}462 \text{ people}}{4.78 \text{ mi}^2} \approx \boxed{3{,}025.5 \text{ people/mi}^2}$$

b. How much difference would adding 1000 people do to the density?

$$d = \frac{15{,}462 \text{ people}}{4.78 \text{ mi}^2} \approx 3{,}234.7 \text{ people/mi}^2$$

It would increase the density by adding over 200 people to each square mile!

c. How many people would need to move out to reduce the density to only 1500 people/mi^2?

$$\frac{1500 \text{ ppl/mi}^2}{1} = \frac{x}{4.78 \text{ mi}^2}$$

$$x = 7{,}170 \text{ people}$$

$$\begin{array}{r} 14{,}462 \\ -\ 7{,}170 \\ \hline 7{,}292 \end{array}$$

7,292 people would need to move!

Example 4: About 70% of earth is covered in water, making 30% of earth potentially habitable by humans. Earth can be modeled as a sphere with diameter 12,700 km. If there are 7.1 billion people in the world, what is the population density of potentially habitable earth? radius = 6,350 km.

$$SA = 4\pi r^2 = 4\pi(6{,}350)^2 \approx 506{,}707{,}479.1 \text{ km}^2$$

$$\begin{array}{r} 506{,}707{,}479.1 \text{ km}^2 \\ \times\ 0.30 \\ \hline 152{,}012{,}243.7 \text{ km}^2 \end{array}$$

$$d = \frac{7{,}100{,}000{,}000 \text{ people}}{152{,}012{,}243.7 \text{ km}^2} \approx \boxed{46.7 \text{ people/km}^2}$$