


# Integrated Math III

# Final Exam Review Packet

## Semester 1 – 2019

**Name** ________________________________

# Semester 1 Formulas

**ALL formulas below will be given within appropriate questions on the final exam**

**Formulas:**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

*Area of a Triangle:* $A = \frac{1}{2} \cdot b \cdot h$

*Area of a Rectangle/Square:* $A = b \cdot h$

*Area of a Parallelogram:* $A = b \cdot h$

*Area of a Rhombus:* $A = b \cdot h$ or $A = \frac{1}{2} \cdot d_1 \cdot d_2$

*Area of a Kite:* $A = \frac{1}{2} \cdot d_1 \cdot d_2$

*Area of a Trapezoid:* $A = \frac{1}{2} \cdot h(b_1 + b_2)$

$(x - h)^2 + (y - k)^2 = r^2$ $\quad (x - h)^2 = 4p(y - k)$ $\quad (y - k)^2 = 4p(x - h)$

| **Formulas:** | | | | | |
|---|---|---|---|---|---|
| Prism | $V = Bh$ | Cone | $V = \frac{1}{3}\pi r^2 h$ | $SA = \pi r^2 + \pi r l$ |
| Cylinder | $V = \pi r^2 h$ | Sphere | $V = \frac{4}{3}\pi r^3$ | $SA = 4\pi r^2$ |
| Pyramid | $V = \frac{1}{3}Bh$ | | | |

Use the space below to list any other formulas you might need (that will not be given to you).

# Unit 1: Analytical Geometry

1. For a circle with center $(5, -2)$ and a point at $(-3, 8)$ ...

   a. Determine the radius.

   $r = \sqrt{(-3-5)^2 + (8-(-2))^2}$

   $= \sqrt{(-8)^2 + (10)^2}$

   $= \sqrt{64 + 100}$

   $= \boxed{\sqrt{164}}$

   b. Write the equation of the circle.

   $(x-h)^2 + (y-k)^2 = r^2$

   $\boxed{(x-5)^2 + (y+2)^2 = 164}$

2. Given the line $y = \frac{4}{3}x - 5$, write the equation of a line perpendicular to the given line that goes through the point $(6, -1)$ in point-slope form.

   opposite reciprocal slopes

   $y - y_1 = m(x - x_1)$

   $m = -\frac{3}{4}$

   $\boxed{y + 1 = \frac{-3}{4}(x-6)}$

3. Given the line $y = \frac{4}{3}x - 5$, write the equation of a line parallel to the given line that goes through the point $(6, -1)$ in slope-intercept form.

   Same Slope

   $y = mx + b$

   $m = \frac{4}{3}$

   $y + 1 = \frac{4}{3}(x - 6)$

   $y + 1 = \frac{4}{3}x - 8$

   $-1 \qquad -1$

   $\boxed{y = \frac{4}{3}x - 9}$

4. Given the equation of a circle is $x^2 + y^2 - 8x + 6y + 16 = 0$, find each of the following:

$-16 \quad -16$

a. The equation in standard form

$x^2 - 8x + \underline{16} + y^2 + 6y + \underline{9} = -16 + \underline{16} + \underline{9}$

$\boxed{(x-4)^2 + (y+3)^2 = 9}$

b. The center $\boxed{(4,-3)}$

c. The radius $\sqrt{9} = \boxed{3}$

c. The diameter $\boxed{6 \text{ units}}$

d. The area

$A = \pi r^2$

$= \pi (3)^2$

$= \boxed{9\pi \text{ units}^2}$

d. The circumference of the circle

$C = 2\pi r$

$= 2\pi(3)$

$= \boxed{6\pi \text{ units}}$

e. Justify if $(-3, 5)$ is on the circle

$(-3-4)^2 + (5+3)^2 = 9$

$(-7)^2 + (8)^2 = 9$

$49 + 64 \neq 9$

$\boxed{(-3,5) \text{ is not on the circle.}}$

5. Given the equation of the parabola: $(x + 2)^2 = 8(y + 2)$, provide the missing information, then graph.

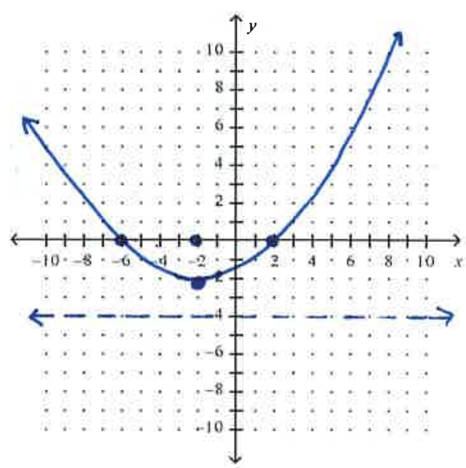
$x^2, + \Rightarrow$ Opens up!

a. Vertex $\underline{(-2,-2)}$

b. P-value $\underline{\frac{8}{4} = 2}$

c. Focus $\underline{(-2,0)}$

d. Directrix $\underline{y = -4}$




6. Given the equation of the parabola: $(y + 1)^2 = -12(x - 3)$, provide the missing information, then graph.

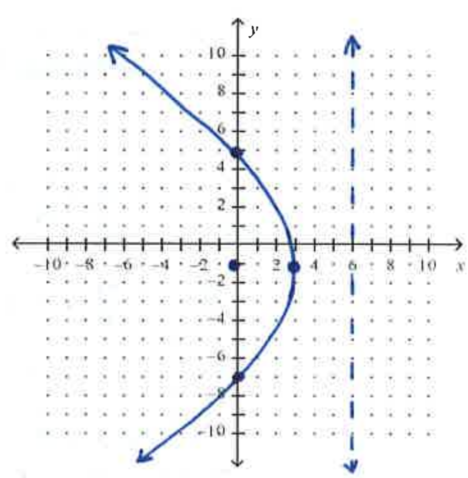
$y^2, - \Rightarrow$ opens left!

a. Vertex $\underline{(3,-1)}$

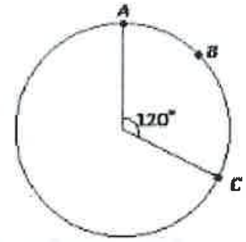
b. P-value $\underline{\frac{12}{4} = 3}$

c. Focus $\underline{(0,-1)}$

d. Directrix $\underline{x = 6}$



7. Using the sector below with a radius of 5 cm.




a. Find the length of the minor arc $\widehat{AC}$.

$\frac{\theta}{360°} = \frac{S}{2\pi r}$ $\quad \frac{120°}{360°} = \frac{S}{2\pi(5)}$ $\quad \frac{360S}{360} = \frac{1200\pi}{360}$

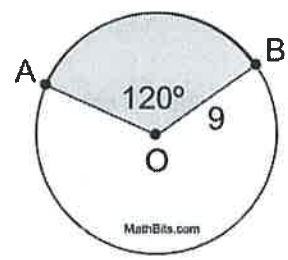
$\boxed{S = \frac{10\pi}{3} \text{ cm.}}$

b. Find the area of the sector created by minor arc $\widehat{AC}$.

$\frac{\theta}{360°} = \frac{A}{\pi r^2}$ $\quad \frac{120°}{360°} = \frac{A}{\pi(5)^2}$ $\quad \frac{360A}{360} = \frac{3000\pi}{360}$

$\boxed{A = \frac{25\pi}{3} \text{ cm}^2}$

8. Using the sector below with a radius of 9 cm.




a. Find the length of the minor arc $\widehat{AB}$.

$\frac{120°}{360°} = \frac{S}{2\pi(9)}$ $\quad \frac{2160\pi}{360} = \frac{360S}{360}$

$\boxed{S = 6\pi \text{ cm.}}$

b. Find the area of the sector created by minor arc $\widehat{AB}$.

$\frac{120°}{360°} = \frac{A}{\pi(9)^2}$ $\quad \frac{360A}{360} = \frac{9720\pi}{360}$

$\boxed{A = 27\pi \text{ cm}^2}$

9. Jose owns a plot of land in the shape of a quadrilateral. If the outside of this plot of land has endpoints of $A: (-3,5), B: (4,5), C: (2,-2)$, and $D: (-5,-2)$.

a. Classify quadrilateral ABCD. **Justify your reasoning using slope and distance.**

It's a $\boxed{\text{parallelogram!}}$

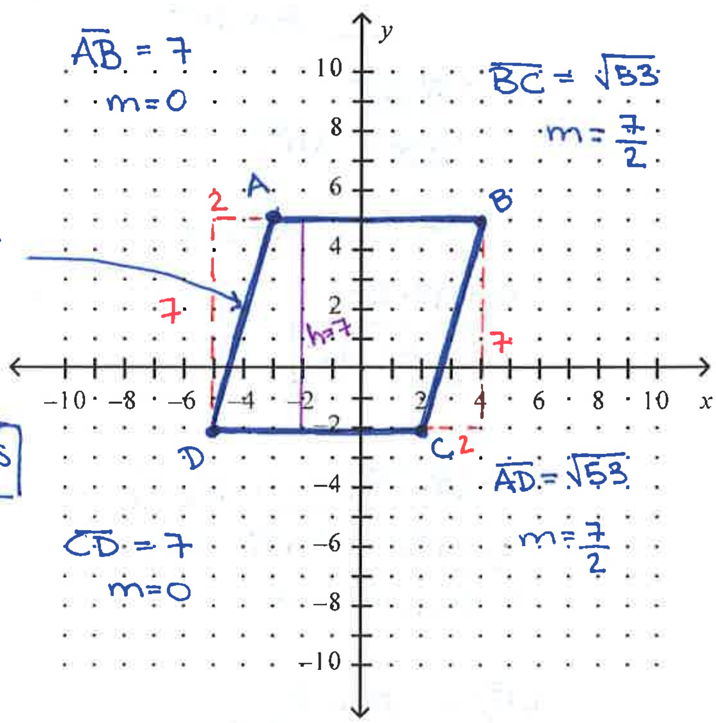
- Opposite sides are parallel because they have the same slope
- Opposite sides are congruent
- No right angles because the slopes aren't opposite reciprocals.

$7^2 + 2^2 = c^2$

$49 + 4 = c^2$

$53 = c^2$

$c = \sqrt{53}$




b. Find the perimeter.

$7 + \sqrt{53} + 7 + \sqrt{53} = \boxed{14 + 2\sqrt{53} \text{ units}}$

c. Determine the area.

$A = b \cdot h = 7 \cdot 7 = \boxed{49 \text{ units}^2}$

# Unit 2: Geometric Proofs and Modeling

10. What are the cross sections formed by each of the following shapes?

a. Cylinder

*Vertical:* rectangle

*Horizontal:* circle

b. Cone

*Vertical:* triangle, $\triangle$

*Horizontal:* circle

c. Sphere:

*Vertical:* circle

*Horizontal:* circle

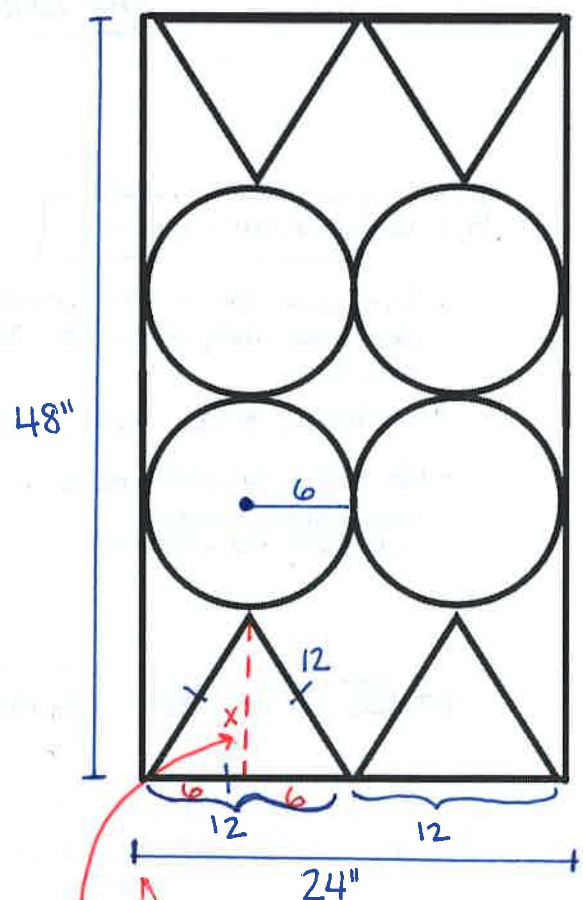
11. Determine which 3-dimensional shape is formed by each of the following.

a. A rectangle rotated around a segment Cylinder

b. A right triangle rotated around a segment Cone

c. A circle rotated along its diameter Sphere

12. I purchased an area rug that measures 24" by 48". The design within the rug contained 4 circles with radius of 6" and 4 equilateral triangles with lengths of 12". Find each of the following: **(not drawn to scale)**




a. Area of one triangle $A=\frac{1}{2}\cdot b\cdot h$

$A=\frac{1}{2}\cdot 12\cdot \sqrt{108}$

$=\boxed{6\sqrt{108} \text{ or } 36\sqrt{3} \text{ in}^2}$

b. Area of one circle $A=\pi r^2$

$A=\pi(6)^2$

$=\boxed{36\pi \text{ in}^2}$

c. Total area of rug $A=b\cdot h$

$A=48\cdot 24$

$=\boxed{1{,}152 \text{ in}^2}$

d. Total area of all triangles and circles

$4\cdot(36\sqrt{3})+4\cdot(36\pi)$

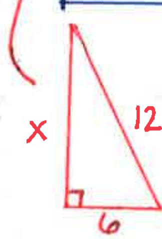
$=\boxed{144\sqrt{3}+144\pi \text{ in}^2}$

$\approx 701.80 \text{ in}^2$

e. Area of the rug not containing triangles or circles.

$1{,}152-701.80$

$\approx\boxed{450.20 \text{ in}^2}$




$x^2+6^2=12^2$

$x^2=108$

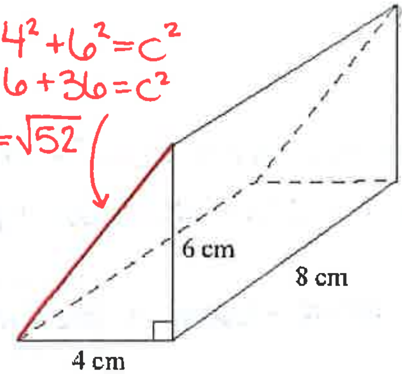
$x=\sqrt{108}$ or $6\sqrt{3}$

13. Use the triangular prism to calculate:

$4^2 + 6^2 = c^2$

$16 + 36 = c^2$

$C = \sqrt{52}$




a. Total Surface Area

front + back: $A = \frac{1}{2}bh = \frac{1}{2}(4)(6) = 12\text{ cm}^2 \times 2 = 24\text{ cm}^2$

bottom: $A = bh = 4(8) = 32\text{ cm}^2$

right side: $A = bh = 6(8) = 48\text{ cm}^2$

left side: $A = bh = 8(\sqrt{52}) = 8\sqrt{52}\text{ cm}^2$

$SA = 24 + 32 + 48 + 8\sqrt{52} = \boxed{104 + 8\sqrt{52}\text{ cm}^2}$

$\approx 161.69\text{ cm}^2$

b. Volume

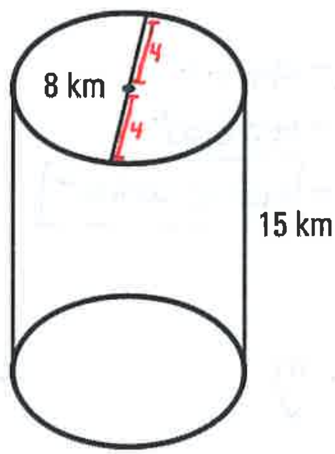
area of the base (Δ)

$V = B \cdot h$

$= \left[\frac{1}{2}(4)(6)\right] \cdot 8$

$= \boxed{96\text{ cm}^3}$

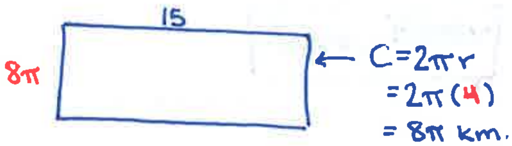
14. Use the diagram to calculate:




a. Total Surface Area

top + bottom: $A = \pi r^2 = \pi(4)^2 = 16\pi\text{ km}^2 \times 2 = 32\pi\text{ km}^2$

Side: $A = bh = 8\pi(15) = 120\pi\text{ km}^2$




$C = 2\pi r$

$= 2\pi(4)$

$= 8\pi\text{ km}$

$SA = 32\pi + 120\pi = \boxed{152\pi\text{ km}^2}$

b. Volume

$V = \pi r^2 h$

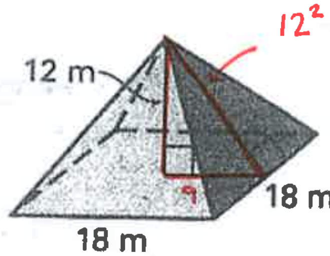
$= \pi(4)^2 \cdot 15$

$= \boxed{240\pi\text{ km}^3}$

15. Use the square pyramid to calculate:

$12^2 + 9^2 = c^2$

$c = 15$




a. Total Surface Area

bottom: $A = bh = 18(18) = 324\text{ m}^2$

Sides: $A = \frac{1}{2}bh = \frac{1}{2}(18)(15) = 135\text{ m}^2 \times 4 = 540\text{ m}^2$

$SA = 324 + 540 = \boxed{864\text{ m}^2}$

b. Volume

$V = \frac{1}{3}Bh$

$= \frac{1}{3}(18 \cdot 18) \cdot 12$

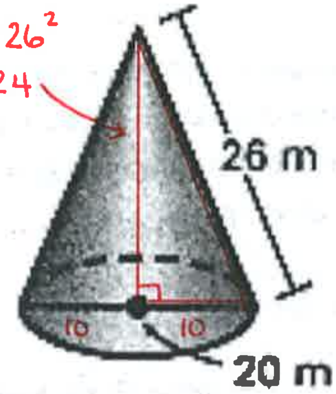
$= \boxed{1{,}296\text{ m}^3}$

16. Use the cone to calculate:

a. Total Surface Area

$10^2+b^2=26^2$

$b=24$




bottom: $A=\pi r^2=\pi(10)^2=100\pi \text{ m}^2$

Side: $A=\pi r l=\pi(10)(26)=260\pi \text{ m}^2$

$SA=100\pi+260\pi=\boxed{360\pi \text{ m}^2}$

b. Volume

$V=\frac{1}{3}\pi r^2 h$

$=\frac{1}{3}\pi(10)^2\cdot 24$

$=\boxed{800\pi \text{ m}^3}$

17. A sphere with a radius of 10.

a. Total Surface Area

$SA=4\pi r^2$

$=4\pi(10)^2$

$=\boxed{400\pi \text{ units}^2}$

b. Volume

$V=\frac{4}{3}\pi r^3$

$=\frac{4}{3}\pi(10)^3$

$=\boxed{\frac{4000\pi}{3} \text{ units}^3}$

18. The average television has a mass of 12,473.79 grams. Find the density if the volume of an average television is 1,507.92 cubic inches.

$d=\frac{m}{V}$

$d=\frac{12{,}473.79 \text{ g.}}{1{,}507.92 \text{ in}^3}$

$\approx \boxed{8.272 \text{ g/in}^3}$

19. The population density of Wrap, IL is 4,564 people per square mile. How many people live in Wrap if the town covers 9.71 mi$^2$

$\frac{4{,}564 \text{ people}}{1 \text{ mi}^2}=\frac{x}{9.71 \text{ mi}^2}$

$x=9.71(4{,}564)\approx\boxed{44{,}316 \text{ people}}$

20. The population density of Hot Dog, IL is 5,155 people per square mile. If there are 16,887 people living outside the bun, how many square miles is Hot Dog, IL?

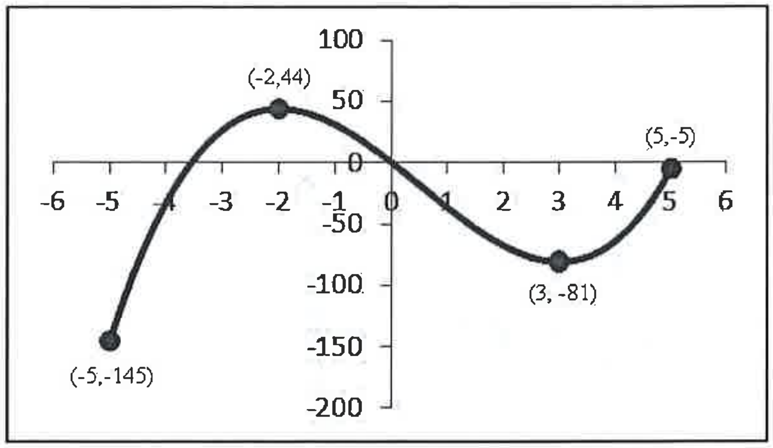
$\frac{5{,}155 \text{ people}}{1 \text{ mi}^2}=\frac{16{,}887 \text{ people}}{x}$

$\frac{\cancel{5{,}155}x}{\cancel{5{,}155}}=\frac{16{,}887}{5{,}155}$

$\boxed{x\approx 3.28 \text{ mi}^2}$

# Unit 3: Representing Functions

21. Use the graph to identify the following:




Relative minimum(s): (-5,-145), (3,-81)

Relative maximum(s): (-2,44), (5,-5)

Absolute maximum(s): (-2, 44)

Absolute minimum(s): (-5, -145)

Increasing interval(s): (-5,-2) and (3,5)

Decreasing interval(s): (-2,3)

Domain: [-5,5]

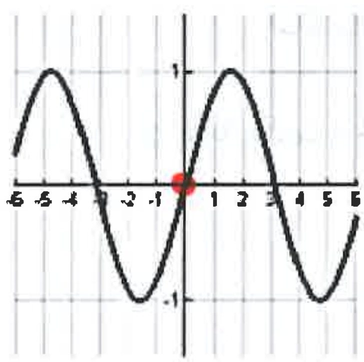
Range: [-145, 44]

X-Intercept(s): (-3.5,0), (0,0)

Y- Intercept(s): (0,0)

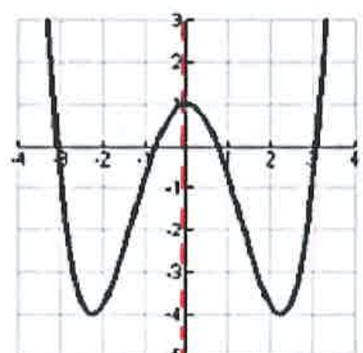
22. Use the graphs below to identify the type of symmetry. Draw the line or point where the graph is symmetric.

a.



Odd-Symmetric about the origin

b.



Even-Symmetric over the y-axis

23. Given the functions $f(x) = x^2$ and $g(x) = -3(x+1)^2 + 4$, determine how each number transforms the new graph **(be specific).**

a. $-3$

- Reflects over the x-axis
- Vertical stretch by a factor of 3

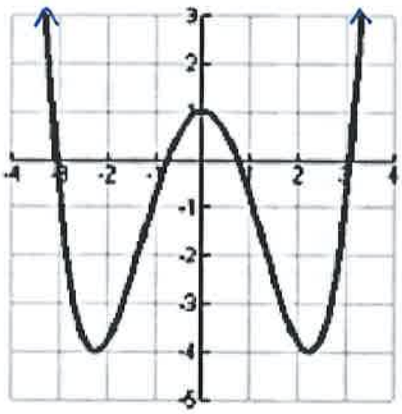
b. 1

left 1 unit

c. 4

up 4 units

24. Use the graph below to identify the indicated key features.




Type of symmetry: Even

Relative maximum(s): $(0, 1)$

Relative minimum(s): $(-2.3, -4)$ and $(2.3, -4)$

Absolute maximum(s): none

Absolute minimum(s): $(-2.3, -4)$ and $(2.3, -4)$

Intervals of increasing: $(-2.3, 0)$ and $(2.3, \infty)$

Intervals of decreasing: $(-\infty, -2.3)$ and $(0, 2.3)$

End behavior: As $x \rightarrow \infty, f(x) \rightarrow \infty$

As $x \rightarrow -\infty, f(x) \rightarrow \infty$

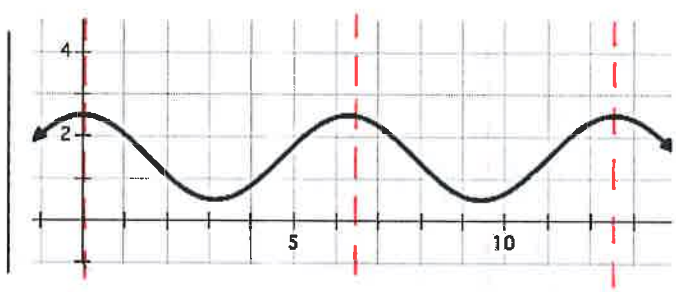
Domain: $(-\infty, \infty)$

Range: $[-4, \infty)$

X-Intercept(s): $(-3,0)$, $(-0.9,0)$, $(0.9,0)$, $(3,0)$

Y- Intercept(s): $(0,1)$

25. Using the following graph, determine if it demonstrates periodicity. If so, what is the period?




It's Periodic!

Period = 6.5 units

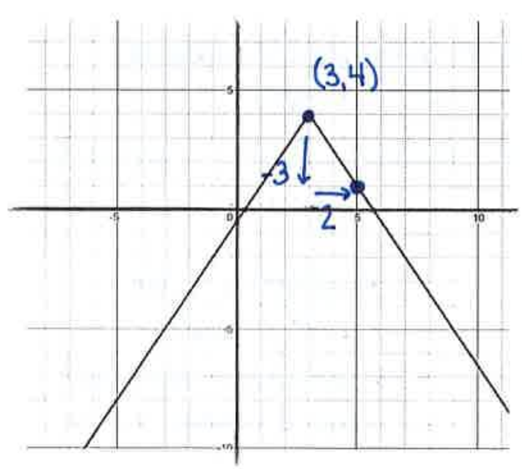
26. Use the graph to answer the questions below.

a. Describe the transformations from the parent graph to the provided one.

- reflected over the x-axis
- vertical stretch by a factor of $3/2$
- right 3 units
- up 4 units

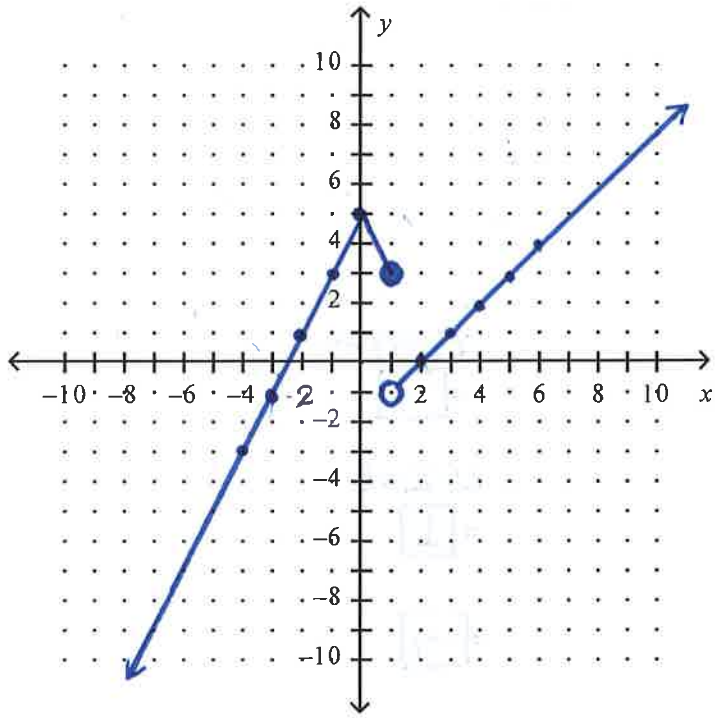
b. Write the equation for this graph.

$y = -\frac{3}{2}|x-3|+4$



27. Graph the following piecewise function. Then, answer the questions that follow.

$$f(x) = \begin{cases} -2|x| + 5, & if\ x \le 1 \\ x - 2, & if\ x > 1 \end{cases}$$



a. Find $f(-4) = -2|-4|+5$

$= \boxed{-3}$

b. Find $f(-2) = -2|-2|+5$

$= \boxed{1}$

c. Find $f(0) = -2|0|+5$

$= \boxed{5}$

d. Find $f(2) = (2)-2$

$= \boxed{0}$

e. Find $f(4) = (4)-2$

$= \boxed{2}$

$-2|x|+5$

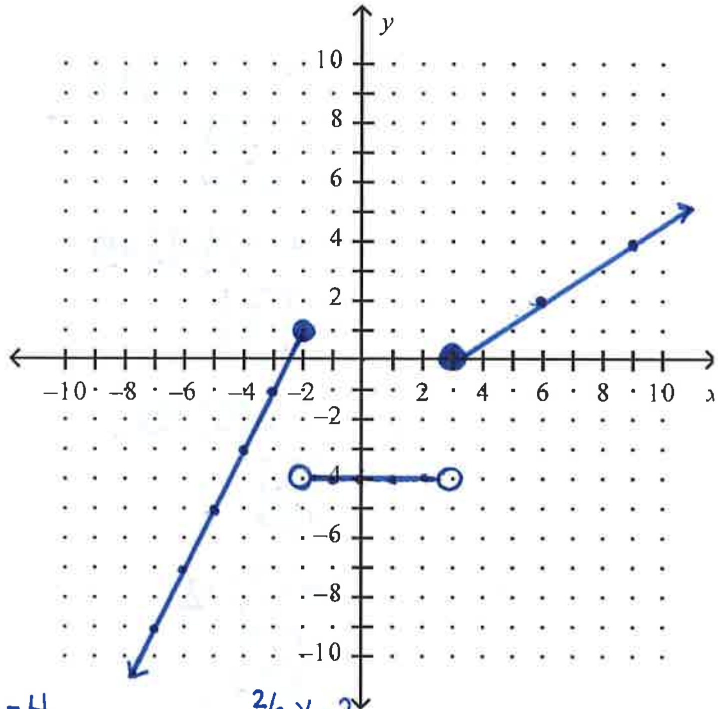
| | | |
|---|---|---|
| closed 1 | $-2\|1\|+5=3$ | (1,3) |
| 0 | $-2\|0\|+5=5$ | (0,5) |
| -1 | $-2\|-1\|+5=3$ | (-1,3) |
| -2 | $-2\|-2\|+5=1$ | (-2, 1) |
| -3 | $-2\|-3\|+5=-1$ | (-3,-1) |
| -4 | $-2\|-4\|+5=-3$ | (-4,-3) |

$x-2$

| | | |
|---|---|---|
| open 1 | $(1)-2=-1$ | (1,-1) |
| 2 | $(2)-2=0$ | (2,0) |
| 3 | $(3)-2=1$ | (3,1) |
| 4 | $(4)-2=2$ | (4,2) |
| 5 | $(5)-2=3$ | (5,3) |
| 6 | $(6)-2=4$ | (6,4) |

28. Graph the following piecewise function. Then, answer the questions that follow.

$$f(x) = \begin{cases} 2x+5\,, & if\ x \le -2 \\ -4\,, & if\ -2 < x < 3 \\ \frac{2}{3}x - 2\,, & if\ x \ge 3 \end{cases}$$



a. Find $f(-4)$

$2(-4)+5$

$= \boxed{-3}$

b. Find $f(-2)$

$2(-2)+5$

$= \boxed{1}$

c. Find $f(0)$

$= \boxed{-4}$

d. Find $f(2)$

$= \boxed{-4}$

e. Find $f(3)$

$\frac{2}{3}(3)-2$

$= \boxed{0}$

f. Find $f(6)$

$\frac{2}{3}(6)-2$

$= \boxed{2}$

$2x+5$

| | | |
|---|---|---|
| Closed | -2 | 1 |
| | -3 | -1 |
| | -4 | -3 |
| | -5 | -5 |
| | -6 | -7 |
| | -7 | -9 |
| | -8 | -11 |

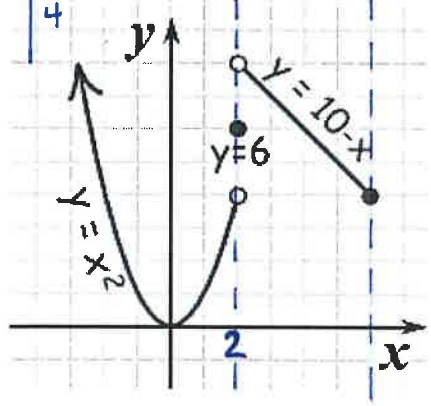
$-4$

| | | |
|---|---|---|
| open | -2 | -4 |
| | -1 | -4 |
| | 0 | -4 |
| | 1 | -4 |
| | 2 | -4 |
| open | 3 | -4 |

$2/3\,x - 2$

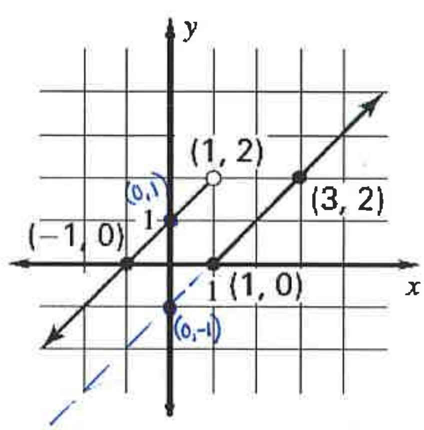
| | | |
|---|---|---|
| closed | 3 | 0 |
| | 4 | 2/3 |
| | 5 | 4/3 |
| | 6 | 2 |
| | 7 | 8/3 |
| | 8 | 10/3 |
| | 9 | 4 |

29. Write a piecewise function for the following graph:




$$f(x) = \begin{cases} x^2\,, & if\ \underline{x<2} \\ 6\,, & if\ \underline{x=2} \\ 10-x\,, & if\ \underline{2<x\le 6} \end{cases}$$

30. Write a piecewise function for the following graph:




$$f(x) = \begin{cases} x+1 & if\ x<1 \\ x-1 & if\ x\ge 1 \end{cases}$$

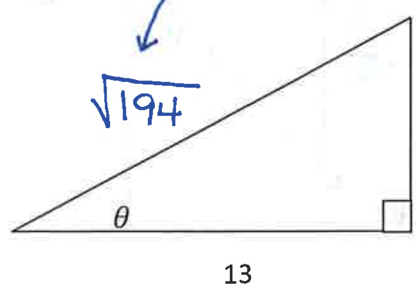
# Unit 4: Trigonometric Representations & Modeling

31. Give the EXACT value for the six trigonometric values of $\theta$.

$5^2 + 13^2 = c^2$

$25 + 169 = c^2$

$194 = c^2$




sin θ = $\frac{5}{\sqrt{194}} = \boxed{\frac{5\sqrt{194}}{194}}$ sec θ = $\boxed{\frac{\sqrt{194}}{13}}$

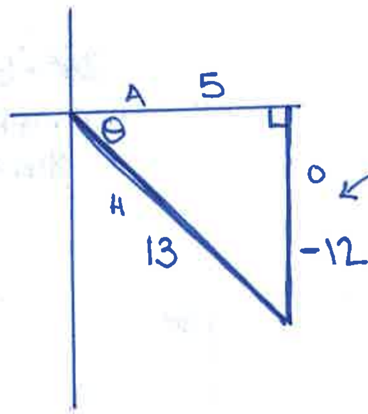
cos θ = $\frac{13}{\sqrt{194}} = \boxed{\frac{13\sqrt{194}}{194}}$ csc θ = $\boxed{\frac{\sqrt{194}}{5}}$

tan θ = $\boxed{\frac{5}{13}}$ cot θ = $\boxed{\frac{13}{5}}$

32. Given that $\theta$ lies in quadrant IV and cos θ = $\frac{5}{13}$ $\frac{A}{H}$, find...

a. sin θ = $\boxed{\frac{-12}{13}}$

b. tan $\theta$ = $\boxed{\frac{-12}{5}}$




$5^2 + b^2 = 13^2$

$25 + b^2 = 169$

$b^2 = 144$

$b = 12$

33. Convert 130° from degrees to radians.

$\frac{130°}{1} \times \frac{\pi}{180°} = \frac{130\pi}{180} = \boxed{\frac{13\pi}{18}}$

*Find the reference angle in degrees* $\boxed{50°}$

$180° - 130° = 50°$

*Find a positive coterminal angle in degrees:* $\boxed{490°}$

$130° + 360° = 490°$

*Find a negative coterminal angle in degrees:* $\boxed{-230°}$

$130° - 360° = -230°$

34. Convert $\frac{4\pi}{5}$ from radians to degrees.

$\frac{4\pi}{5} \times \frac{180°}{\pi} = \frac{720\cancel{\pi}}{5\cancel{\pi}} = \boxed{144°}$

*Find the reference angle in degrees* $\boxed{36°}$

$180° - 144° = 36°$

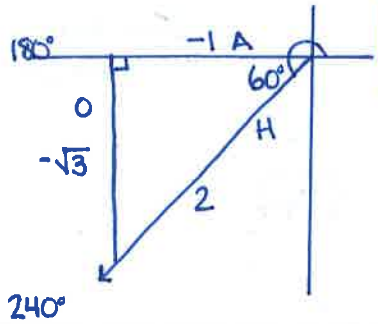
*Find a positive coterminal angle in degrees:* $\boxed{504°}$

$144° + 360° = 504°$

*Find a negative coterminal angle in degrees:* $\boxed{-216°}$

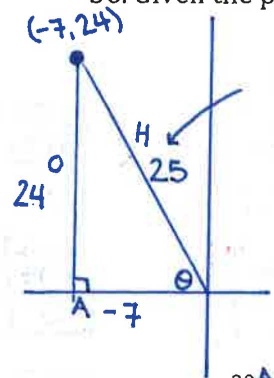
$144° - 360° = -216°$

35. Find the EXACT value (no decimals) of cos 240°.




$\boxed{\cos 240° = \frac{-1}{2}}$

36. Given the point $(-7, 24)$, determine cos θ.




$(-7)^2 + 24^2 = c^2$

$49 + 576 = c^2$

$625 = c^2$

$\boxed{\cos\theta = \frac{-7}{25}}$

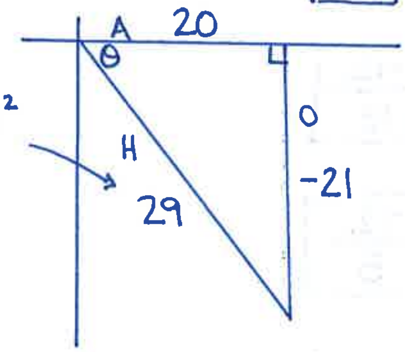
37. Find the exact value (no calculator) for the following trig functions if cot $= -\frac{20}{21}$ and the terminal side is in quadrant IV.

sin θ = $\boxed{\frac{-21}{29}}$ cos θ = $\boxed{\frac{20}{29}}$ tan θ = $\boxed{\frac{-21}{20}}$

$20^2 + (-21)^2 = c^2$

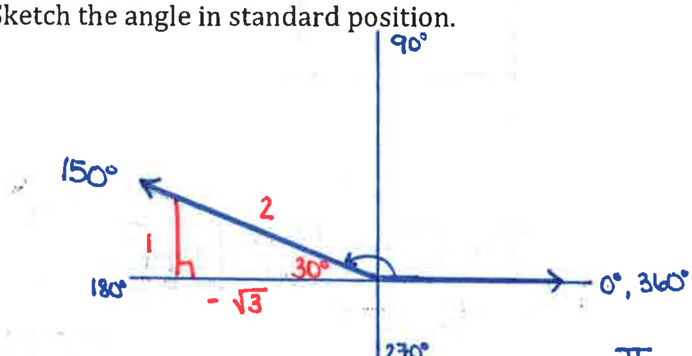
$400 + 441 = c^2$

$841 = c^2$




38. Given an angle with measure 150°…

a. Sketch the angle in standard position.




b. Determine the quadrant the terminal side lies in. II

c. Find the measure of the reference angle. 30°

$180° - 150° = 30°$

d. Draw a reference triangle & evaluate all 6 trig functions.

sin 150° = $\boxed{\frac{1}{2}}$ cos 150° = $\boxed{\frac{-\sqrt{3}}{2}}$ tan 150° = $-\frac{1}{\sqrt{3}} = \boxed{\frac{-\sqrt{3}}{3}}$

csc 150° = $\frac{2}{1} = \boxed{2}$ sec 150° = $\frac{-2}{\sqrt{3}} = \boxed{\frac{-2\sqrt{3}}{3}}$ cot 150° = $\frac{-\sqrt{3}}{1} = \boxed{-\sqrt{3}}$

39. Simplify: $\sin^2\theta - 3 + \cos^2\theta$

∗ $\sin^2\theta + \cos^2\theta = 1$

$= 1 - 3$

$= \boxed{-2}$

40. Verify: $\sin\theta \cot\theta = \cos\theta$

$\frac{\cancel{\sin\theta}}{1} \cdot \frac{\cos\theta}{\cancel{\sin\theta}} = \cos\theta$

$\cos\theta = \cos\theta$ ✓

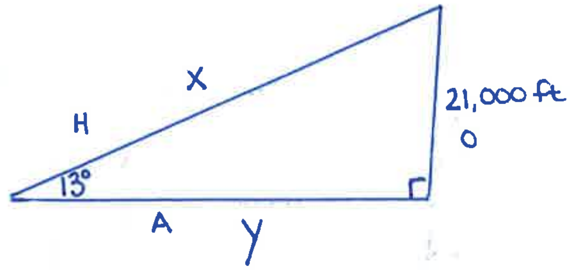
41. As I prepared to come home from Charleston, SC, my family waited for me at the airport. They noticed that my plane was 21,000 feet above the ground, the angle of elevation was 13°.

a. How far was I still away from my family?

$\cancel{\frac{\sin 13°}{1} = \frac{21{,}000}{x}}$

$\frac{x \sin 13°}{\sin 13°} = \frac{21{,}000}{\sin 13°}$

$\boxed{x \approx 93{,}353.64 \text{ ft}}$




b. What is the horizontal distance that I was from the airport?

$\cancel{\frac{\tan 13°}{1} = \frac{21{,}000}{y}}$

$\frac{y \tan 13°}{\tan 13°} = \frac{21{,}000}{\tan 13°}$

$\boxed{y \approx 90{,}960.99 \text{ ft}}$

## Unit 5: Graphing and Modeling Trigonometric Equations

42. What is the period of the graph represented by the equations below? Period $= \frac{2\pi}{b}$

a. $f(x) = \cos(4x)$ $b = 4$

$\frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$

b. $f(x) = 3\sin\left(\frac{1}{2}x\right)$ $b = 1/2$

$\frac{2\pi}{1/2} = \boxed{4\pi}$

c. $f(x) = 4\cos(x)$ $b = 1$

$\frac{2\pi}{1} = \boxed{2\pi}$

43. What is the amplitude of graph represented by the equations below?

a. $f(x) = 4\sin(3x) + 5$

$\boxed{\text{Amp} = 4}$

b. $f(x) = \cos(x) - 1$

$\boxed{\text{Amp} = 1}$

c. $f(x) = -3\sin 4x$

$\boxed{\text{Amp} = 3}$

44. What is the horizontal shift of the equation $y = -2\cos(x + \pi) - 1$? → phase shift

Phase Shift $= \frac{-c}{b} = \frac{-\pi}{1} = \boxed{-\pi}$

45. What is the vertical shift of the equation $y = 2\sin(x - \pi) + 3$?

$\boxed{y = 3}$

46. Given the function $y = 2\sin(x) - 3$, fill in the following blanks and graph:

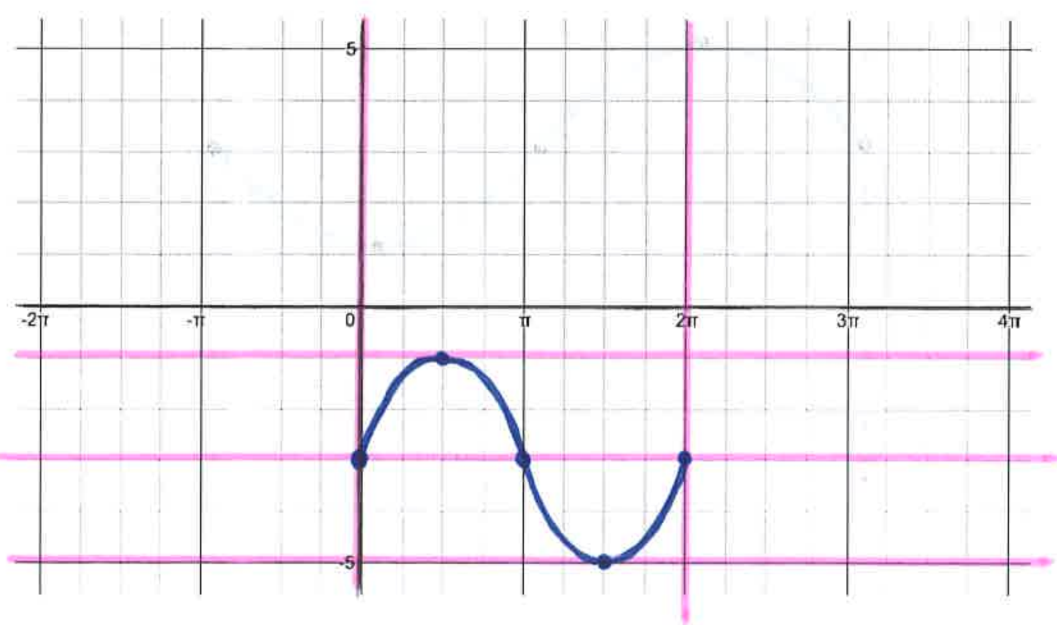
Amplitude: 2

Vertical Shift: $y = -3$

Period: $\frac{2\pi}{1} = 2\pi$

Phase/Horizontal Shift: Ø

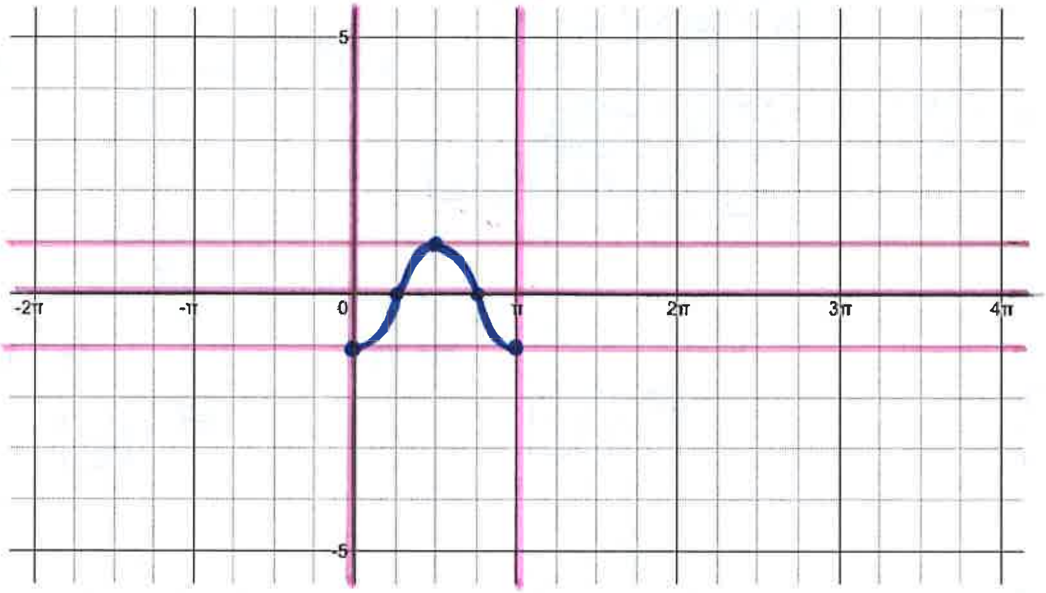
Range: $[-5, -1]$



47. Given the function $y = -\cos(2x)$, fill in the following blanks and graph:

Amplitude: 1 Vertical Shift: $\varnothing$ Period: $\frac{2\pi}{2} = \boxed{\pi}$

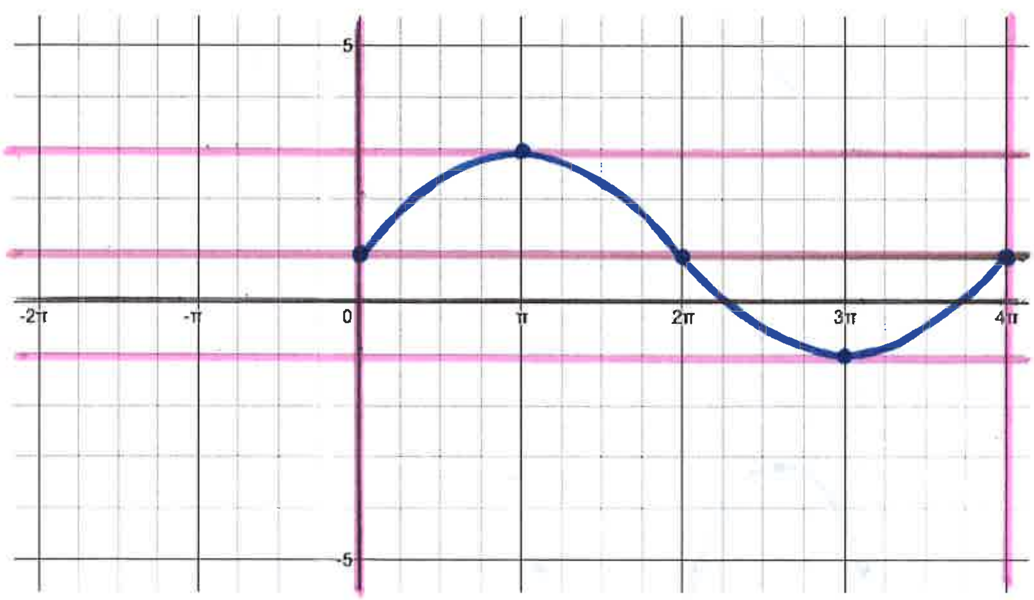
Phase/Horizontal Shift: $\varnothing$ Range: $[-1, 1]$



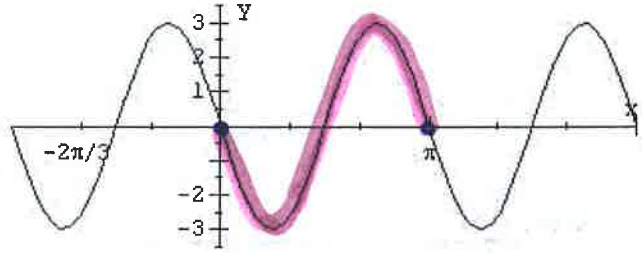

48. Given the function $= 2\sin\left(\frac{1}{2}x\right) + 1$, fill in the following blanks and graph:

Amplitude: 2 Vertical Shift: $y=1$ Period: $\frac{2\pi}{1/2} = \boxed{4\pi}$

Phase/Horizontal Shift: $\varnothing$ Range: $[-1, 3]$



49. Using the graph to the right, identify the:




a. Amplitude $= \boxed{3}$

b. Period $= \pi - 0 = \boxed{\pi}$

c. Horizontal shift $= \boxed{\text{NONE}}$

d. Vertical Shift $= \boxed{\text{NONE}}$

e. Range $= \boxed{[-3, 3]}$

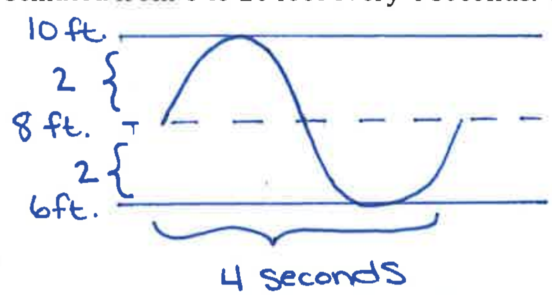
f. Equation $\boxed{y = -3\sin(2x)}$

Period $= \frac{2\pi}{b}$

$\cancel{\frac{\pi}{1} = \frac{2\pi}{b}}$

$\frac{b\pi}{\pi} = \frac{2\pi}{\pi}$ $b = 2$

50. A fisherman noticed that the height of a bobber oscillated from 6 to 10 feet every 4 seconds. Identify each of the following




a. Midline $\boxed{y = 8}$

b. Amplitude $= \boxed{2 \text{ feet}}$

c. Period $= \boxed{4 \text{ seconds}}$

d. Equation (express this function in terms of sine)

$\boxed{y = 2\sin\left(\frac{\pi}{2}x\right) + 8}$

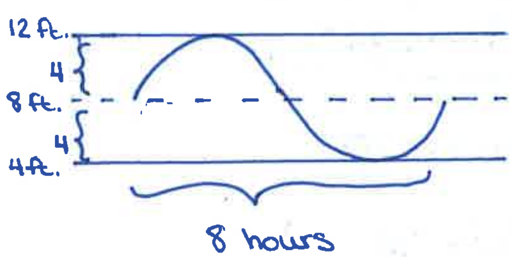
Period $= \frac{2\pi}{b}$

$\cancel{\frac{4 \text{ seconds}}{1} = \frac{2\pi}{b}}$

$b = \pi/2$

51. Write an equation to best represents the following situations:

a. The time between high tides is 8 hours (period). The high-tide depth is 12 feet and the low-tide depth is 4 feet.




$\boxed{y = 4\sin\left(\frac{\pi}{4}x\right) + 8}$

Period $= \frac{2\pi}{b}$

$\cancel{\frac{8 \text{ hours}}{1} = \frac{2\pi}{b}}$

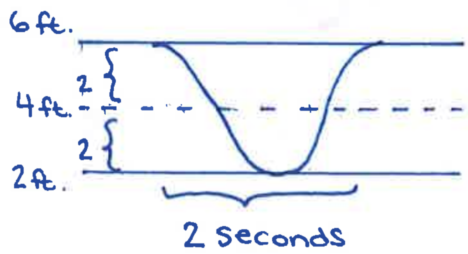
$\frac{8b}{8} = \frac{2\pi}{8}$

$b = \frac{\pi}{4}$

cosine!

b. Mrs. Kelleher is pushing her daughter on a swing. When she starts the swing at its highest point, her daughter is 6 feet above the ground. At the lowest point, the swing is 2 feet above the ground. It takes approximately 2 seconds for the swing to leave and come back to Mrs. K.

period




$$\boxed{y=2\cos(\pi x)+4}$$

$$\text{Period} = \frac{2\pi}{b}$$

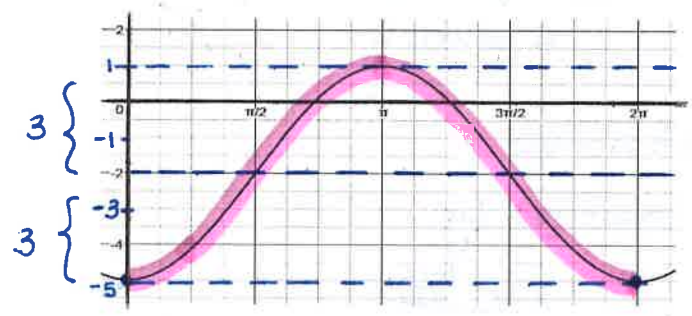
$$\frac{2\text{ seconds}}{1} = \frac{2\pi}{b}$$

$$\frac{\cancel{2}b}{\cancel{2}} = \frac{\cancel{2}\pi}{\cancel{2}}$$

$$b=\pi$$

52. Write the equation that corresponds with each graph below.

a.




$$\boxed{y=-3\cos(x)-2}$$

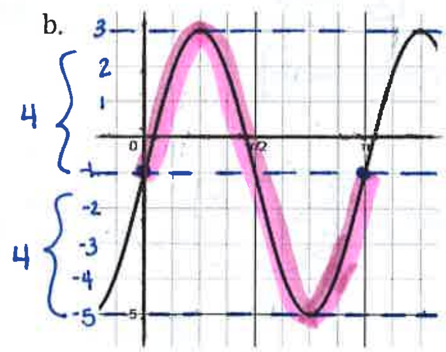
$$\text{Period} = \frac{2\pi}{b}$$

$$\frac{2\pi}{1} = \frac{2\pi}{b}$$

$$\frac{\cancel{2\pi}b}{\cancel{2\pi}} = \frac{\cancel{2\pi}}{\cancel{2\pi}}$$

$$b=1$$

b.




$$\boxed{y=4\sin(2x)-1}$$

$$\text{Period} = \frac{2\pi}{b}$$

$$\frac{\pi}{1} = \frac{2\pi}{b}$$

$$\frac{\cancel{\pi}b}{\cancel{\pi}} = \frac{2\cancel{\pi}}{\cancel{\pi}}$$

$$b=2$$