Integrated Math 3
Unit 3: Representing Functions
3.4

Name: ______________________

Date:__________ Period:______

Part I Objective: To discover patterns in graphing.

Warm up: What is the difference between even and odd functions?

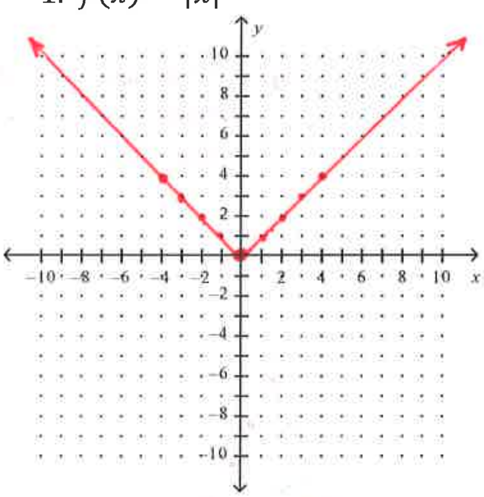
Even functions are symmetric about the y-axis whereas odd functions are symmetric about the origin.

Using your graphing calculator, graph each of the following equations – you can get the absolute value on your calculator by pressing the MATH key, then arrow over to NUM and your first choice should be abs(then just press ENTER. **Be sure to close your parenthesis when the absolute value ends.**

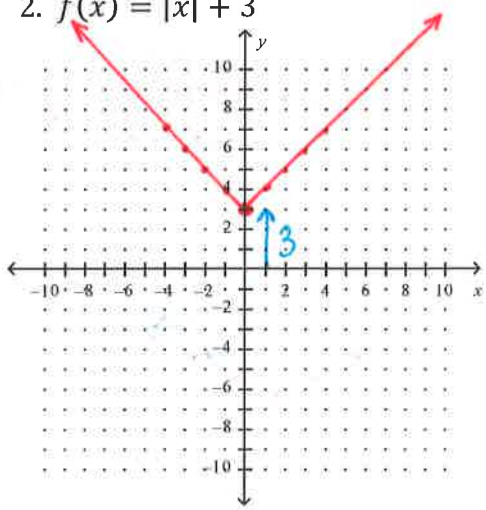
Let's look at what happens when we add a number to the outside of the function:

Parent function

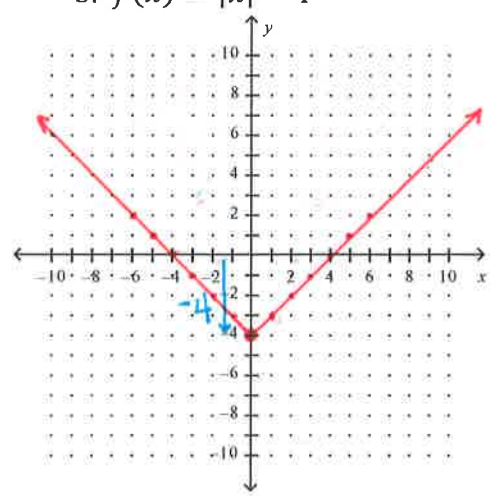
1. $f(x) = |x|$



2. $f(x) = |x| + 3$ ← up 3



3. $f(x) = |x| - 4$ ↙ down 4

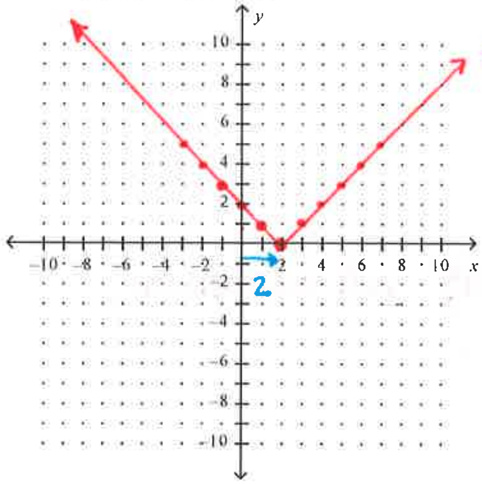


What happens to the graph if you add a number to the outside of the function? Be very specific in how it changes based off of the provided equation.

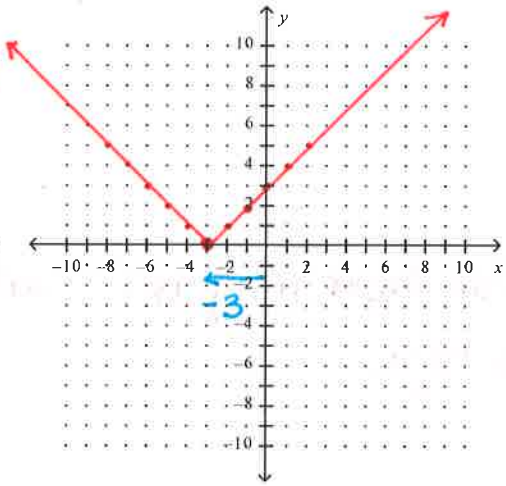
It applies a vertical shift - if we're adding a value outside the function, it moves it up that many units. If we're subtracting a value outside the function, it moves it down that many units.

Let's look at what happens when we add a number to the inside of the function:

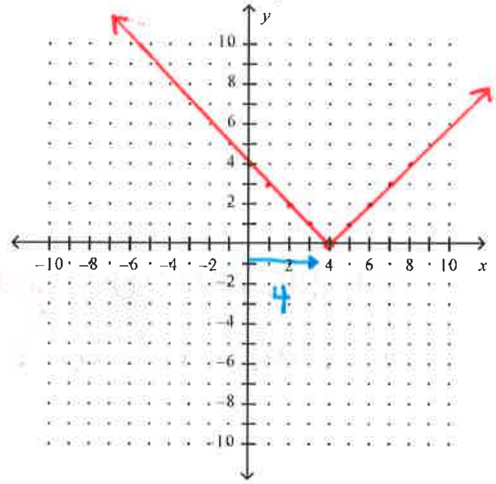
4. $f(x) = |x - 2|$ → right 2



5. $f(x) = |x + 3|$ → left 3



6. $f(x) = |x - 4|$ → right 4



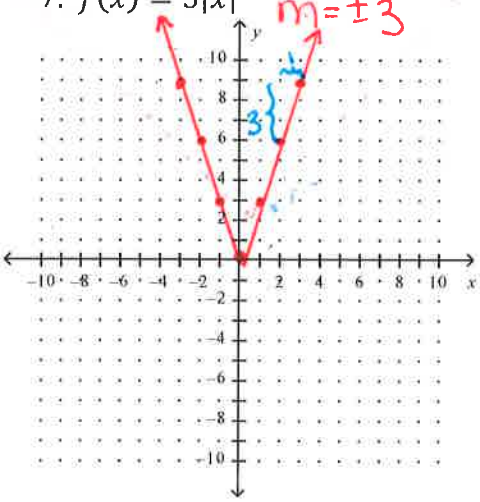
What happens to the graph if you add a number to the inside of the function? Be very specific in how it changes based off of the provided equation.

It applies a horizontal shift - we always do the opposite of the sign when it's inside

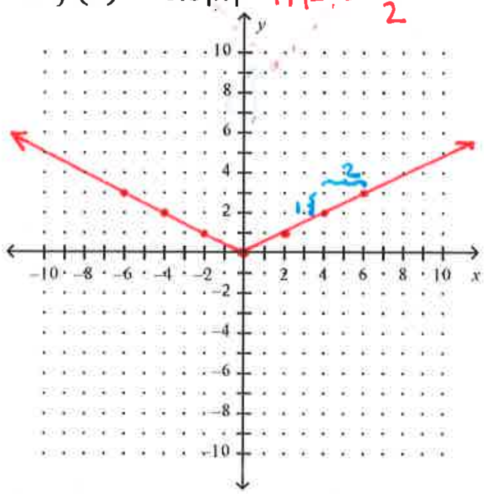
- If adding, move to the left that many units
- If subtracting, move to the right that many units.

Let's look at what happens when we multiply a number to the function:

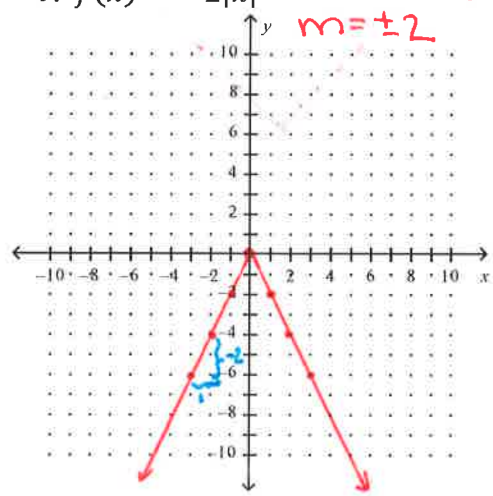
7. $f(x) = 3|x|$ → stretches by 3 $m = \pm 3$



8. $f(x) = 0.5|x|$ → narrows by 0.5 $m = \pm \frac{1}{2}$



9. $f(x) = -2|x|$ → reflects over x-axis stretches by 2 $m = \pm 2$



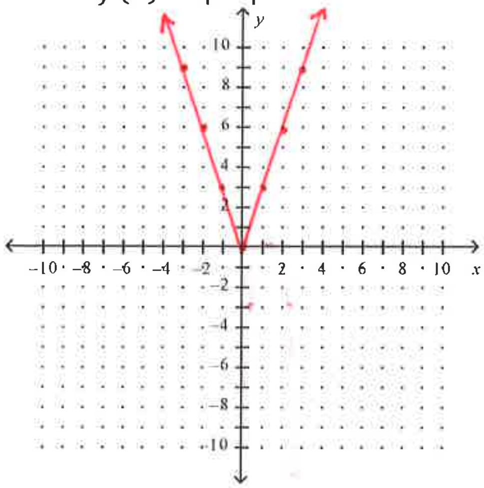
What happens to the graph when you multiply a number to the function? Be very specific in how it changes based off of the provided equation.

It determines the slope or steepness of a function and the direction of opening

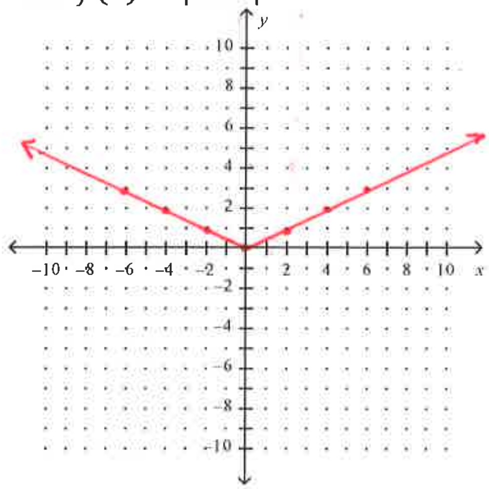
- If we're multiplying by a negative, it opens down; if multiplying by a positive, opens up
- If it's bigger than 1, it stretches (or narrows) the graph; if smaller than 1, it compresses (or widens) the graph in the y-direction

Let's look at what happens when we multiply a number within the function:

10. $f(x) = |3x|$

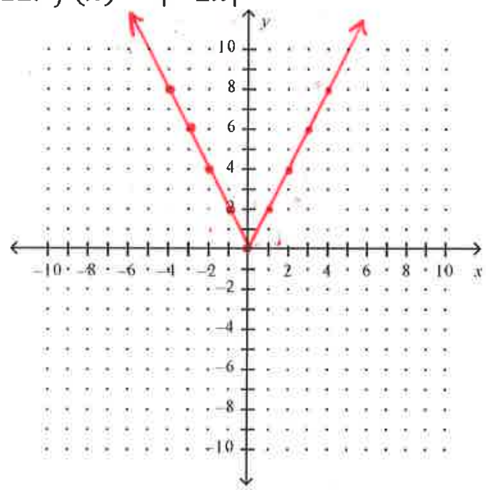


11. $f(x) = |0.5x|$



Reflects over y-axis

12. $f(x) = |-2x|$



What happens to the graph when you multiply a number within the function? Be very specific in how it changes based off of the provided equation.

If the number is bigger than 1, it compresses it in the x-direction

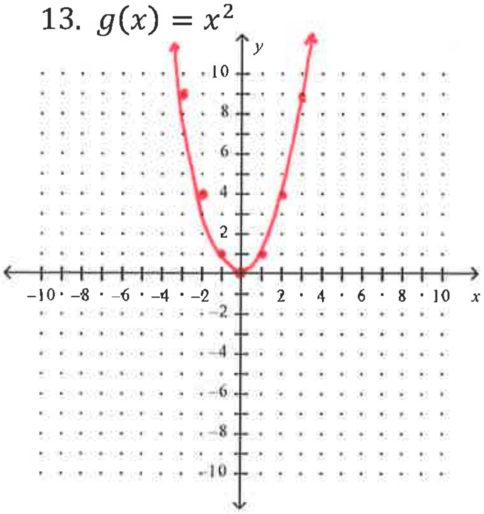
If the number is smaller than 1, it stretches it in the x-direction

How do graphs #7-9 compare to graphs #10-12? Why do you think that happened?

They're identical.

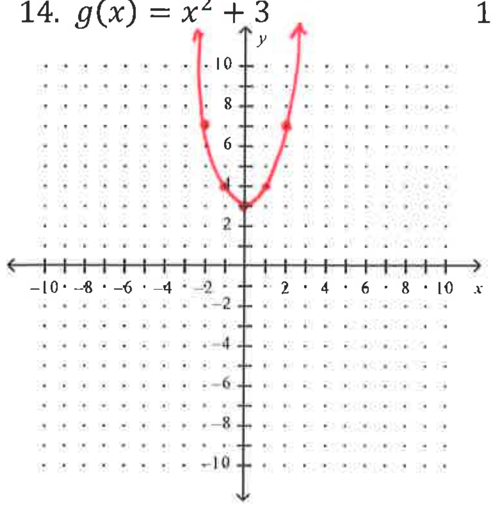
Let's see if this is true for quadratic equations...

13. $g(x) = x^2$



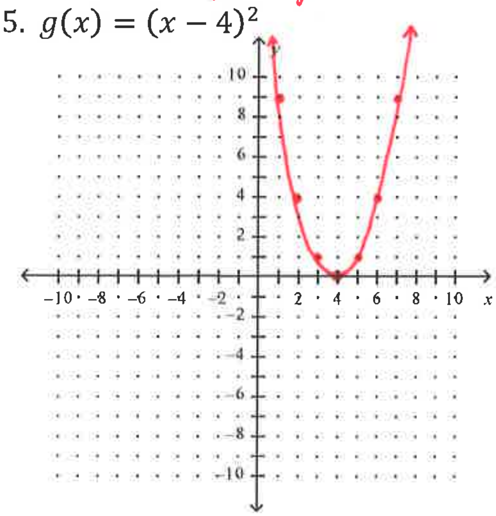
up 3

14. $g(x) = x^2 + 3$



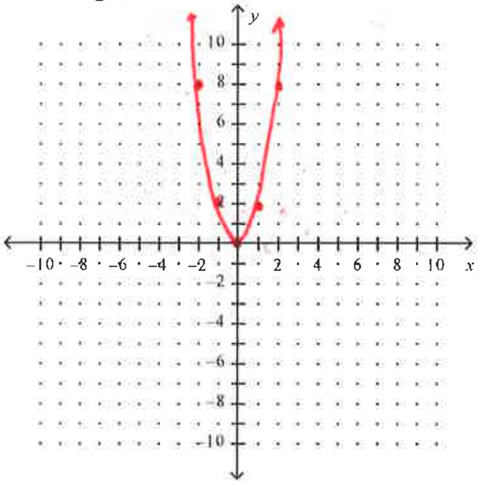
right 4

15. $g(x) = (x-4)^2$

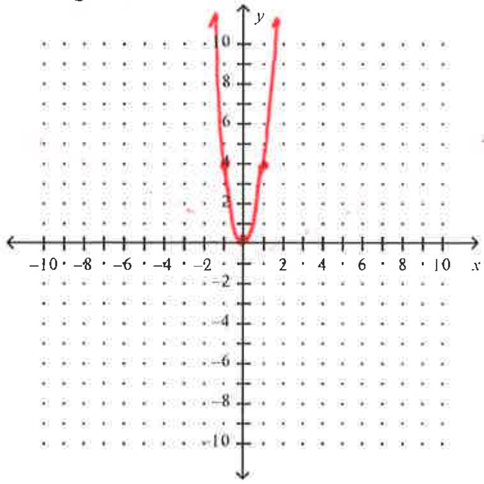


Let's see if this is true for <u>quadratic</u> equations...

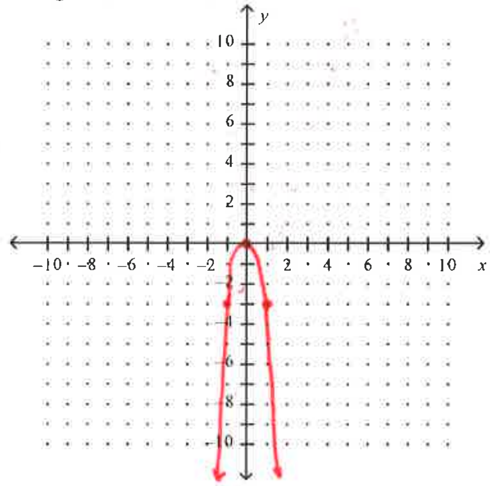
16. $g(x) = 2x^2$



17. $g(x) = (2x)^2$



18. $g(x) = -3(x)^2$



Explain the similarities and differences in the patterns between absolute value functions and quadratic functions. If there are none, write none.

	Similarities	**Diferences**
$f(x) = \lvert x \rvert$ and $g(x) = x^2$	end behavior always points in the same direction on both sides vertex is at (0,0)	absolute value function is formed by straight lines and has a constant slope whereas the quadratic is a rounded graph whose rate changes
$f(x) = \lvert x \rvert + k$ and $g(x) = x^2 + k$		
$f(x) = \lvert x + k \rvert$ and $g(x) = (x + k)^2$		
$f(x) = k\lvert x \rvert$ and $g(x) = kx^2$		
$f(x) = \lvert kx \rvert$ and $g(x) = (kx)^2$		

Part II Objective: Analyzing and writing absolute value functions.

Warm Up:

1. Identify the distance each of the following values is from zero:

6	-6	3	-3	4	-4
6	6	3	3	4	4

2. Identify the center of the circles given their equations.

$(x-3)^2+(y+2)^2=9$

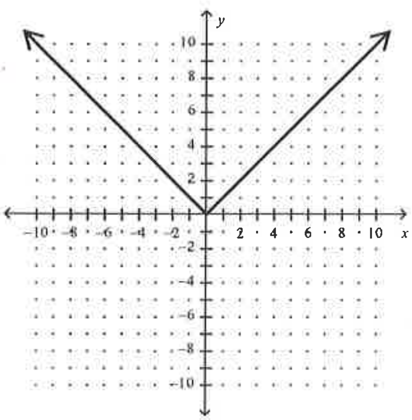
center: $(3,-2)$
radius: 3

$x^2+(y-5)^2=16$

center: $(0,5)$
radius: 4

What is an Absolute Value Function?

An absolute value function, when graphed, is a function consisting of two rays whose ends meet at a common point, called the vertex, and extend into a "v" shape.



$$\boxed{y = a|x-h|+k}$$

a: horizontal stretch/compression (narrow vs. widen)

$|a|>1$ stretches

$|a|<1$ compresses

h, k: vertex: (h, k)

Some Fun Facts About Absolute Value Functions:

- Absolute value functions are Symmetric around the y-axis
- For every point (x, y) existing on the graph, there exists the point $(-x, y)$
- The vertex (h, k) also indicates a horizontal shift and/or a vertical shift
- The coefficient a in front of the absolute value signs indicates the:
  - Slope of the two rays, which describes the overall Steepness of the function
  - direction of opening → if positive, opens up
    → if negative, opens down

Example 1: Identify the vertex, the steepness of the function and the direction of opening given the following absolute value functions:

A.) $y = |x|$

vertex: (0,0)

opens up

B.) $y = 3|x| - 2$

vertex: (0,-2)

opens up

Stretches by 3 in the y-direction

C.) $y = 2|x - 4| + 3$

vertex: (4,3)

opens up

stretches by 2 in the y-direction

D.) $y = |x + 5| + 1$

vertex: (-5,1)

opens up

E.) $y = -|x - 1| + 6$

vertex: (1,6)

opens down

F.) $y = -\frac{1}{2}|x + 2| + 2$

vertex: (-2,2)

opens down

Compresses by 1/2 in the y-direction

Example 2: Write an absolute value function given the following properties.

A.) Has a vertex located at $(-3, 5)$

$y = |x+3| + 5$

B.) Has a vertex located at $(0, -2)$ and opens down

$y = -|x| - 2$

C.) Has a vertex at $(0, 0)$ and has rays with slopes of 3

$y = 3|x|$

D.) Has a vertex at $(-4, 0)$, opens down and has rays with slopes of $\frac{1}{2}$

$y = -\frac{1}{2}|x+4|$

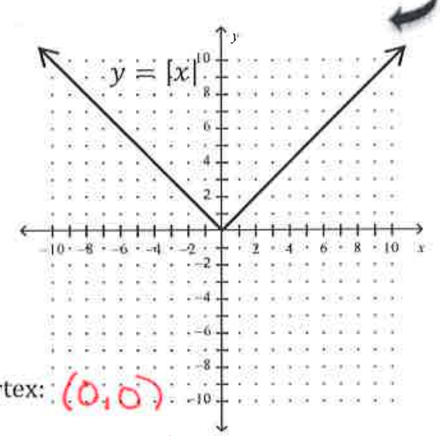
E.) Has a vertex at $(-3, -\frac{1}{2})$ and has rays with slopes of 2

$y = 2|x+3| - \frac{1}{2}$

Example 3: Given the graph of the absolute value function, identify the vertex, direction of opening, and the slopes of the rays. Explain how the graph transformed from the original $y = |x|$ graph. Finally, write an equation representing the graphed function.

The ORIGINAL (parent graph)

A.)

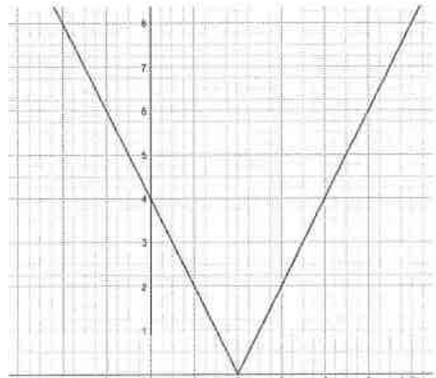


Vertex: (0,0)

Slopes of Rays: ± 1

Direction of Opening: up

B.)



Vertex: (2,0)

Direction of Opening: up

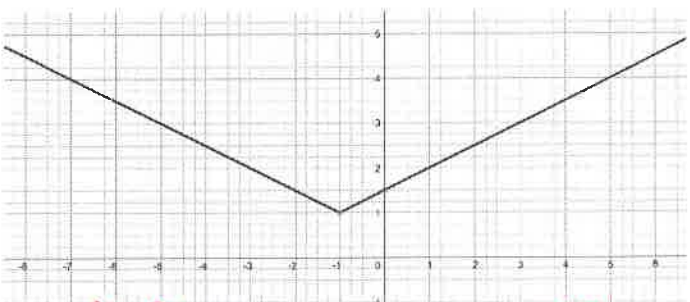
Slopes of Rays: ± 2

Transformations: right 2 stretches by 2 in the y-direction

Equation:

$y = 2|x-2|$

B.)



Vertex: (−1,1)

Direction of Opening: up

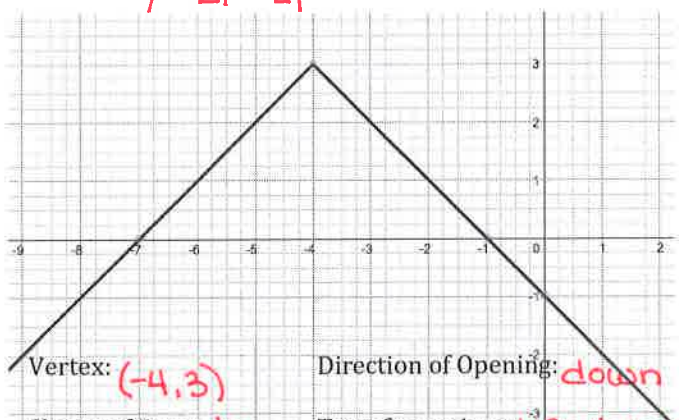
Slopes of Rays: $\pm \frac{1}{2}$

Transformations: left 1, up 1, Compresses by ½ in the y-direction

Equation:

$y = \frac{1}{2}|x+1|+1$

C.)



Vertex: (−4,3)

Direction of Opening: down

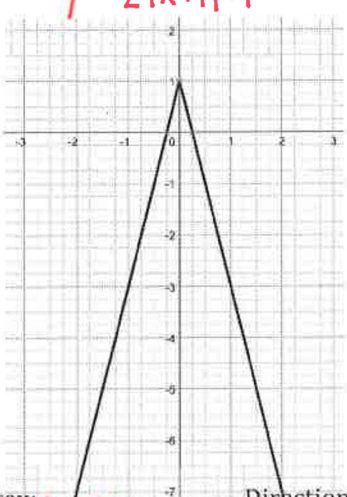
Slopes of Rays: ± 1

Transformations: left 4, up 3, reflect over x-axis

Equation:

$y = |x+4|+3$

D.)



Vertex: (0,1)

Direction of Opening: down

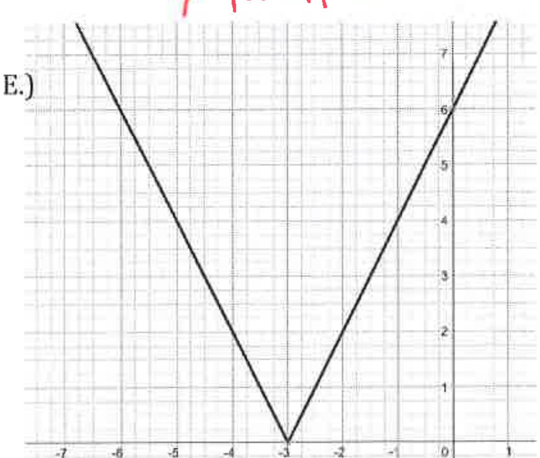
Slopes of Rays: ± 4

Transformations: up 1, reflect over x-axis, stretches by 4 in the y-direction

Equation:

$y = -4|x|+1$

E.)



Vertex: (−3,0)

Direction of Opening: up

Slopes of Rays: ± 2

Transformations: left 3, stretches by 2 in the y-direction.

Equation:

$y = 2|x+3|$