Integrated Math 3
Unit 4: Trig. Representations and Modeling
4.3

Name: _____

Date:_____ Period:_____

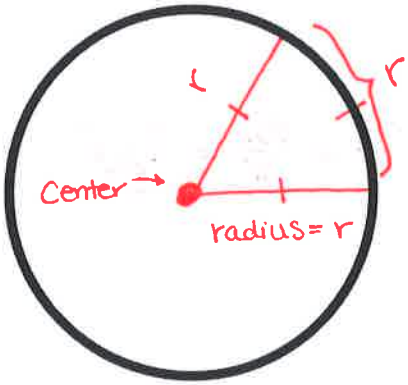
Part I Objective: To convert radian and degree measures.

Warm Up: If there are 5280 feet in a mile, how much of a mile is 218 feet?

$$218 \text{ ft.} \times \frac{1 \text{ mi.}}{5280 \text{ ft.}} = .0413 \text{ mi.}$$

What is a radian?

- A radian is another method of describing an angle measure (previously we only used degrees to measure angles).
- One radian = the measure of an angle whose intercepted arc is the length of the radius.



Having a conversion factor allows us to convert between two units

Background Knowledge:

1. How many degrees are in a circle? 360°
2. How do you find circumference of a circle? $C = 2\pi r$
3. Using the circumference formula above, what is the EXACT circumference of a circle with radius of 1? $C = 2\pi(1) = 2\pi$
4. Therefore we can say 360 degrees = 2π radians $\quad \frac{360°}{2\pi \text{ rad}} = \frac{180°}{\pi \text{ rad}}$

Rewrite a degree measure in radians by multiplying by $\frac{\text{radians}}{180}$
Rewrite a radian measure in degrees by multiplying by $\frac{180}{\text{radians}}$

Example 1: Convert the degree measure to radians.

A.) 110°

$110° \times \frac{\pi}{180°} = \boxed{\frac{11\pi}{18}}$

B.) 45°

$45° \times \frac{\pi}{180} = \boxed{\frac{\pi}{4}}$

C.) 320°

$320° \times \frac{\pi}{180°} = \boxed{\frac{16\pi}{9}}$

D.) 225°

$225° \times \frac{\pi}{180°} = \boxed{\frac{5\pi}{4}}$

E.) 330°

$330° \times \frac{\pi}{180°} = \boxed{\frac{11\pi}{6}}$

F.) −45°

$-45° \times \frac{\pi}{180°} = \boxed{\frac{-\pi}{4}}$

Example 2: Convert the radian measure to degrees.

A.) $-\frac{\pi}{9}$

$\frac{-\pi}{9} \times \frac{180°}{\pi} = \boxed{-20°}$

B.) $-\frac{5\pi}{12}$

$\frac{-5\pi}{12} \times \frac{180°}{\pi} = \boxed{-75°}$

C.) $\frac{28\pi}{3}$

$\frac{28\pi}{3} \times \frac{180°}{\pi} = \boxed{1680°}$

D.) $\frac{2\pi}{3}$

$\frac{2\pi}{3} \times \frac{180°}{\pi} = \boxed{120°}$

E.) $-\frac{3\pi}{2}$

$\frac{-3\pi}{2} \times \frac{180°}{\pi} = \boxed{-270°}$

F.) $\frac{5\pi}{6}$

$\frac{5\pi}{6} \times \frac{180°}{\pi} = \boxed{150°}$

Part II Objective: To analyze coterminal and reference angles.

Vocabulary:

Angle: A shape composed of two rays with a common endpoint, known as the vertex.

Standard Position: An angle whose vertex is at the origin and whose initial side is the positive x-axis.

Initial Side: The ray of the angle that is "fixed".

Terminal Side: The ray of the angles that gets rotated about the vertex.

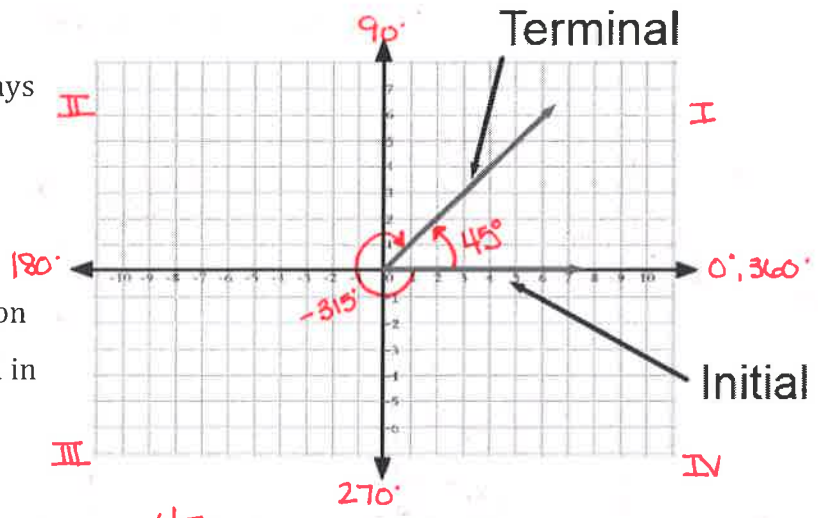
Coterminal: Two angles that are in standard position, whose terminal sides end at the same location.

Reference Angles: An angle (always acute in measure) formed by the terminal side and the x-axis.

Quadrantal Angle: An angle whose terminal side lies on an axis.

Fill in the blank:

Use the graph to complete each of the following



a. The initial side of an angle is always on the positive x-axis

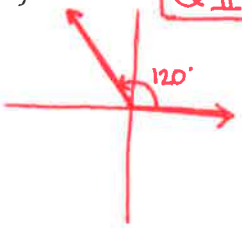
b. When analyzing angles, positive angles are measured in a Counter-Clockwise direction and negative angles are measured in a clockwise direction.

c. The angle above can be estimated to be + 45° or - 315°

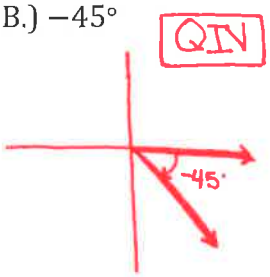
Angles in standard position

Example 1: Draw the following angles in standard position. Then tell which quadrant the terminal side lies in.

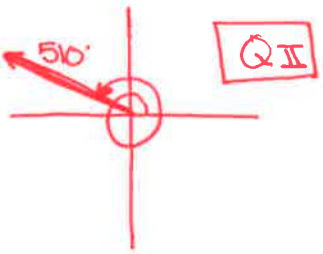
A.) 120° QII



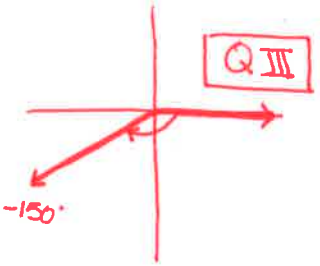
B.) −45° QIV



C.) 510° $-360° = 150°$



D.) $-\frac{5\pi}{6} \times \frac{180°}{\pi} = -150°$



Coterminal Angles

Example 2: Determine the positive and negative coterminal angles of the given angle in degrees and radians.

A.) $120° \times \frac{\pi}{180°} = \frac{2\pi}{3}$

Positive: $120° + 360° = 480°$

$\frac{2\pi}{3} + 2\pi = \frac{8\pi}{3}$

negative: $120° - 360° = -240°$

$\frac{2\pi}{3} - 2\pi = -\frac{4\pi}{3}$

B.) $45° \times \frac{\pi}{180} = \frac{\pi}{4}$

$45° + 360° = 405°$

$\frac{\pi}{4} + 2\pi = \frac{9\pi}{4}$

$45° - 360° = -315°$

$\frac{\pi}{4} - 2\pi = -\frac{7\pi}{4}$

C.) $210° \times \frac{\pi}{180°} = \frac{7\pi}{6}$

$210° + 360° = 570°$

$\frac{7\pi}{6} + 2\pi = \frac{19\pi}{6}$

$210° - 360° = -150°$

$\frac{7\pi}{6} - 2\pi = -\frac{5\pi}{6}$

D.) $\frac{\pi}{6} \times \frac{180°}{\pi} = 30°$

$30° + 360° = 390°$

$\frac{\pi}{6} + 2\pi = \frac{13\pi}{6}$

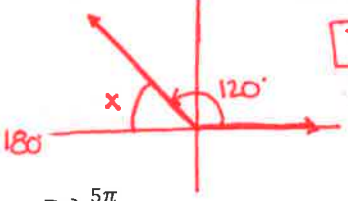
$30° - 360° = -330°$

$\frac{\pi}{6} - 2\pi = -\frac{11\pi}{6}$

Reference Angles – measures how far to the x-axis
– should always be positive!

Example 3: Determine the reference angles for the following:

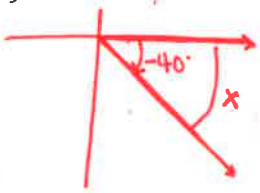
A.) 120°



$x + 120° = 180°$

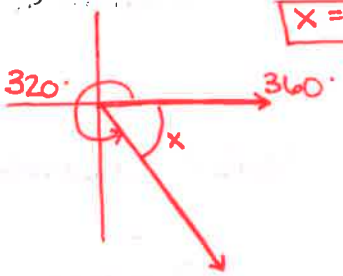
$x = 60°$

B.) −40°



40°

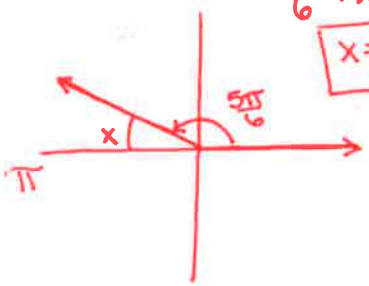
C.) 320°



$320° + x = 360°$

$x = 40°$

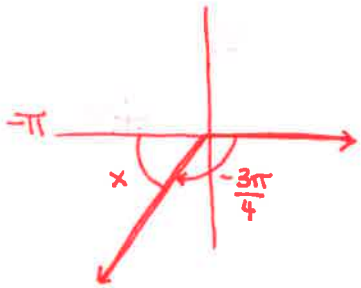
D.) $\frac{5\pi}{6}$



$\frac{5\pi}{6} + x = \pi$

$x = \frac{\pi}{6}$

E.) $-\frac{3\pi}{4}$



$-\frac{3\pi}{4} + x = -\pi$

$x = -\frac{\pi}{4}$

$x = \frac{\pi}{4}$