Integrated Math 3
Unit 6: Polynomials
6.12

Name: ____________________

Date: __________ Period: ______

Objective: To factor a polynomial function using the factor theorem and find the zero of a function.

Warm Up: Given $y = x^2 - 4x + 3$

a. Write the quadratic equation in factored form.

$y = (x-1)(x-3)$

	x	-1
x	x^2	$-x$
-3	$-3x$	3

Mult: $3x^2$
Add: $-4x$

b. Why is the equation in part (a) also called intercept form?

It gives you the values of the x-intercepts – all you need to do is split the factors and set them equal to zero, and then solve for x.

Vocabulary:

Factor Theorem: A polynomial $f(x)$ has a factor $(x-k)$ if and only if $f(k) = 0$. This is the same as saying k is a zero of the function.

Remainder Theorem: If a polynomial $f(x)$ is divided by $(x-k)$, then the remainder is $r = f(k)$.

Example 1: Is $\overset{k=-1}{(x+1)}$ a factor of $x^3 - x^2 + 2$?

$(-1)^3 - (-1)^2 + 2$
$= -1 - 1 + 2$
$= 0$ ✓

Yes, it is a factor.

Example 2: Use the remainder theorem to find the remainder for each division. State whether the binomial is a factor of the polynomial.

a. $(x^2 - x + 4) \div \overset{k=2}{(x-2)}$

$(2)^2 - (2) + 4$
$= 4 - 2 + 4$
$= 6$

Remainder? yes because $6 \neq 0$

factor? no because there's a remainder

b. $(x^3 + x^2 - 17x + 15) \div \overset{k=-5}{(x+5)}$

$(-5)^3 + (-5)^2 - 17(-5) + 15$
$= 0$

remainder? no

factor? yes because there's no remainder

c. $\frac{(x^2+20x+91)}{x+7} \rightarrow K=-7$

$(-7)^2+20(-7)+91$

$=0$

remainder? <u>no</u>

factor? <u>yes</u>

d. $(x^3-9x^2+27x-28)\div(x-3)$ $K=3$

$(3)^3-9(3)^2+27(3)-28$

$=-1$

remainder? <u>yes</u> since $-1\neq 0$

factor? <u>no</u>

<u>Steps to factoring using the factoring theorem</u>

1. Write the function in standard form
2. Set up the area model
3. Find the factor that was used to multiply (x-k) to get the polynomial by working backwards.
4. Identify your answer:
   a) If there is no remainder, k is the x-intercept (and a factor of the polynomial)
   b) If there is a remainder, plug k in to the polynomial to confirm that k is not a factor.

<u>Example 3</u>: Factor using the area model.

a. $f(x) = 2x^3 + 11x^2 + 18x + 9$ when $k = -3$ $(x+3)$

	$2x^2$	$5x$	3
x	$2x^3$	$5x^2$	$3x$
3	$6x^2$	$15x$	9

$f(x) = (x+3)\underbrace{(2x^2+5x+3)}$

Mult: $6x^2$

Add: $5x$

	x	1
$2x$	$2x^2$	$2x$
3	$3x$	3

$\boxed{f(x) = (x+3)(2x+3)(x+1)}$

b. $f(x) = 3x^3 + 13x^2 + 2x - 8$ when $k = -4$ $(x+4)$

	$3x^2$	x	-2
x	$3x^3$	x^2	$-2x$
4	$12x^2$	$4x$	-8

$f(x) = (x+4)\underbrace{(3x^2+x-2)}$

Mult: $-6x^2$

Add: $-x$

	x	-1
$3x$	$3x^2$	$-3x$
2	$2x$	-2

$\boxed{f(x) = (x+4)(3x+2)(x-1)}$

Example 4: Factor and then find all the zeros of the function.

a. $f(x) = x^3 - 2x^2 - 9x + 18$ when $k = 2$ $(x-2)$

	x^2	$0x$	-9
x	x^3	$0x^2$	$-9x$
-2	$-2x^2$	$0x$	18

$f(x) = (x-2)\underbrace{(x^2-9)}_{\text{difference of squares!}}$

$$\boxed{f(x) = (x-2)(x-3)(x+3)}$$

b. $f(x) = x^3 + 6x^2 + 3x - 10$ when $k = -5$ $(x+5)$

	x^2	x	-2
x	x^3	x^2	$-2x$
5	$5x^2$	$5x$	-10

$f(x) = (x+5)\underbrace{(x^2+x-2)}$

Mult: $-2x^2$

Add: x

	x	2
x	x^2	$2x$
-1	$-x$	-2

$$\boxed{f(x) = (x+5)(x-1)(x+2)}$$