Integrated Math 3 Name: ______________________________
Unit 6: Polynomials
6.12 Date: __________ Period: ______

Objective: to use end behavior to create sketches of graphs.

Warm Up:

1. Write $f(x) = \underbrace{6x^2 - 13x - 5}_{-30x^2}$ in complete factored form. What does factored form help you solve for?

$\boxed{(2x-5)(3x+1)}$

	$2x$	-5
$3x$	$6x^2$	$-15x$
1	$2x$	-5

Factored form helps us solve for the zeros / roots / x-intercepts.

2. Write $f(x) = x^3 + 6x^2 - 5x - 30$ in complete factored form. What does factored form help you solve for?

$\boxed{(x^2-5)(x+6)}$

	x	6
x^2	x^3	$6x^2$
-5	$-5x$	-30

Factored form helps us to solve for the zeros / roots / x-intercepts.

3. Identify all of the x-intercepts for #1 & #2. Explain what the x-intercepts represent for a polynomial.

#1)

$(2x-5)(3x+1)=0$

$2x-5=0$ $3x+1=0$

$2x=5$ $3x=-1$

$x=\frac{5}{2}$ $x=\frac{-1}{3}$

x-int: $(\frac{5}{2},0), (-\frac{1}{3},0)$

#2)

$(x^2-5)(x+6)=0$

$x^2-5=0$ $x+6=0$

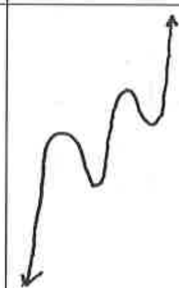
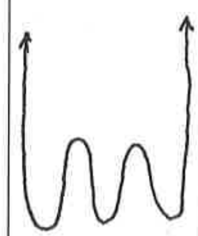
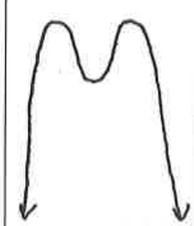
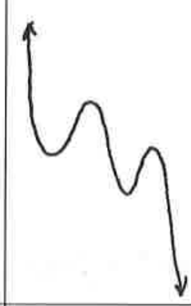
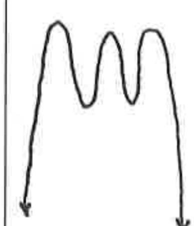
$x^2=5$ $x=-6$

$x=\pm\sqrt{5}$

x-int: $(-\sqrt{5},0), (\sqrt{5},0), (-6,0)$

x-intercepts represent the location where the function touches the x-axis on the graph.

Use either a graphing calculator device or access the website www.desmos.com to explore a trend of what the following polynomials look like. Your task is to find a pattern, or shortcut, in determining a general shape for the polynomials.

CHARACTERISTICS OF POLYNOMIALS		2nd degree	3rd degree	4th degree	5th degree	6th degree
Possible Number of X-intercepts		2	3	4	5	6
General Shape of Polynomial	Positive Leading Coefficient	$f(x)=2x^2+3$ $f(x)=3x^2+2x+1$ $f(x)=5x^2+7x-3$	$f(x)=x^3+4x$ $f(x)=4x^3-2x^2+5x+1$ $f(x)=2x^3+7x^2-4x+3$	$f(x)=x^4+3x^2$	$f(x)=x^5-2x$	$f(x)=x^6-3x^2$
						
	End Behavior	$x\to\infty$, $f(x)\to\infty$ $x\to-\infty$, $f(x)\to\infty$	$x\to\infty$, $f(x)\to\infty$ $x\to-\infty$, $f(x)\to-\infty$	$x\to\infty$, $f(x)\to\infty$ $x\to-\infty$, $f(x)\to\infty$	$x\to\infty$, $f(x)\to\infty$ $x\to-\infty$, $f(x)\to-\infty$	$x\to\infty$, $f(x)\to\infty$ $x\to-\infty$, $f(x)\to\infty$
	Negative Leading Coefficient	$f(x)=-2x^2$ $f(x)=-3x^2+7x+1$ $f(x)=-4x^2+2x-5$	$f(x)=-3x^3$ $f(x)=-2x^3+5x^2-7x+1$ $f(x)=-7x^3+4x^2+5x-2$	$f(x)=-4x^4$	$f(x)=-x^5$	$f(x)=-x^6$
						
	End Behavior	$x\to\infty$, $f(x)\to-\infty$ $x\to-\infty$, $f(x)\to-\infty$	$x\to\infty$, $f(x)\to-\infty$ $x\to-\infty$, $f(x)\to\infty$	$x\to\infty$, $f(x)\to-\infty$ $x\to-\infty$, $f(x)\to-\infty$	$x\to\infty$, $f(x)\to-\infty$ $x\to-\infty$, $f(x)\to\infty$	$x\to\infty$, $f(x)\to-\infty$ $x\to-\infty$, $f(x)\to-\infty$
Maximum # of Turns		1	2	3	4	5

Follow up:

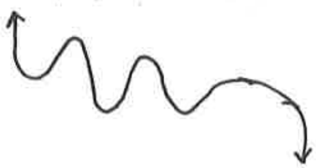
- Make at least two generalizations from the data you collected above

1. If it's an odd degree, the ends point in opposite directions.
    - Positive = right side up
    - Negative = left side up
2. If it's an even degree, the ends point in the same direction.
    - Positive = up
    - Negative = down

- Make a prediction about the general shape of the following two polynomials without looking at a device. After your prediction, confirm with a device that your prediction holds true.

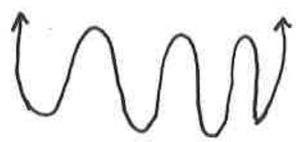
1. $f(x) = -x^{7} + x^3 - 4$ odd

negative



2. $f(x) = x^{8} + 2$ even

positive



Let's Practice!

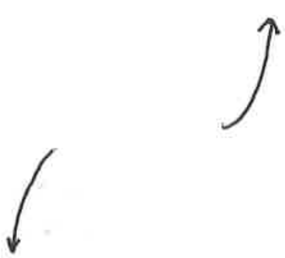
1. Briefly sketch what the following functions could look like. Be sure to identify the degree and leading coefficient first!

A.) $f(x) = x^3 + 4x^2 - 3$

Degree: 3

Sign of Leading Coefficient: +

Sketch:



B.) $f(x) = -x^6 + 2$

Degree: 6

Sign of Leading Coefficient: —

Sketch:

C.) $f(x) = x + x^4 - 3$

Degree: 4

Sign of Leading Coefficient: +

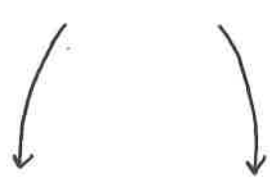
Sketch:

D.) $f(x) = 3x + 5 - 2x^2$

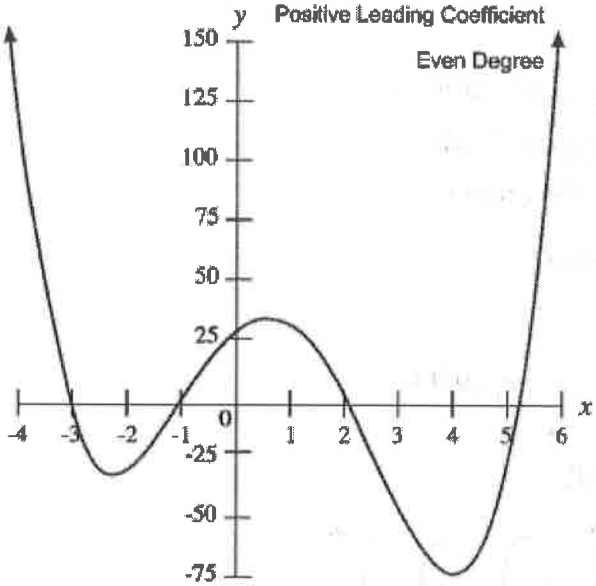
Degree: 2

Sign of Leading Coefficient: —

Sketch:



Example 1: Identify the following about the polynomial graphed below.

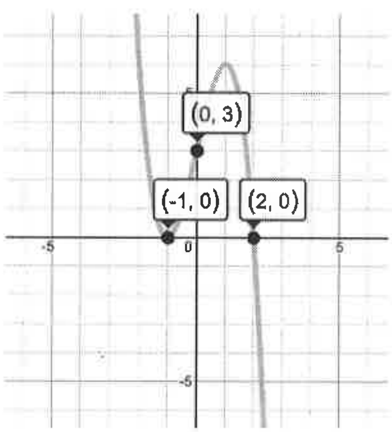


- Describe the end behavior. $x \to \infty, f(x) \to \infty$; $x \to -\infty, f(x) \to \infty$
- Leading Coefficient's Sign $+$
- Degree of Function 4
- X-intercepts $(-3,0)(-1,0)(2.1,0)(5.25,0)$
- Factors $(x+3)(x+1)(x-2.1)(x-5.25)$

Let's Practice!

First write the polynomial in factored form. Then describe the end behavior using limit notation!

A.)



End Behavior: $x \to \infty, f(x) \to -\infty$
$x \to -\infty, f(x) \to \infty$

Leading Coefficient Sign: $-$

Degree of Function: 3

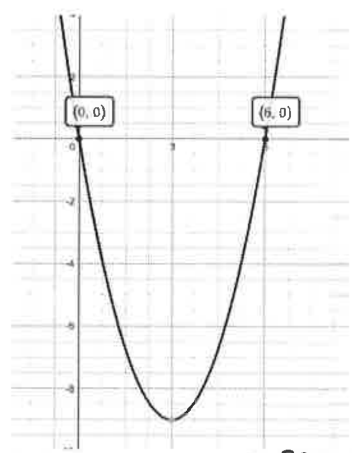
X-intercepts: $(-1,0), (2,0)$

Factors: $(x+1)(x-2)$

Possible Equation:

$f(x) = -(x+1)^2(x-2)$

B.)



End Behavior: $x \to \infty, f(x) \to \infty$
$x \to -\infty, f(x) \to \infty$

Leading Coefficient Sign: $+$

Degree of Function: 2

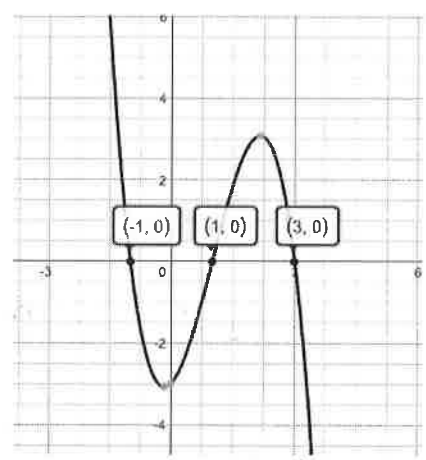
X-intercepts: $(0,0), (6,0)$

Factors: $x(x-6)$

Possible Equation:

$f(x) = x(x-6)$

C.)



End Behavior: $x \to \infty, f(x) \to -\infty$
$x \to -\infty, f(x) \to \infty$

Leading Coefficient Sign: $-$

Degree of Function: 3

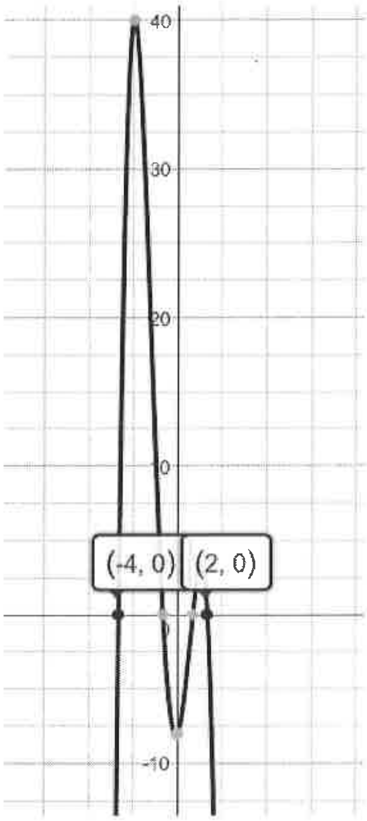
X-intercepts: $(-1,0), (1,0), (3,0)$

Factors: $(x+1)(x-1)(x-3)$

Possible Equation:

$f(x) = -(x+1)(x-1)(x-3)$

D.)



End Behavior: $x \to \infty, f(x) \to -\infty$
$x \to -\infty, f(x) \to -\infty$

Leading Coefficient Sign: —

Degree of Function: 4

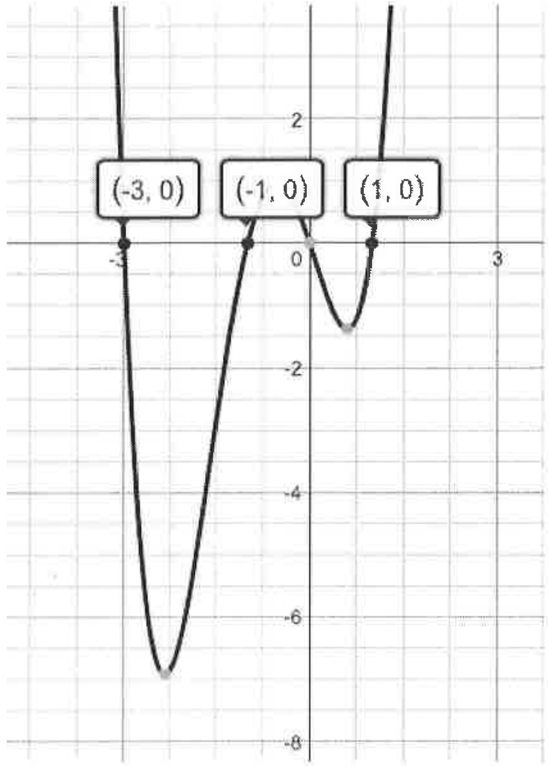
X-intercepts: (-4,0), (2,0), (1,0), (-1,0)

Factors: $(x+4)(x-2)(x-1)(x+1)$

Possible Equation:

$f(x) = -(x+4)(x-2)(x-1)(x+1)$

E.)



End Behavior: $x \to \infty, f(x) \to$
$x \to -\infty, f(x) \to$

Leading Coefficient Sign: +

Degree of Function: 4

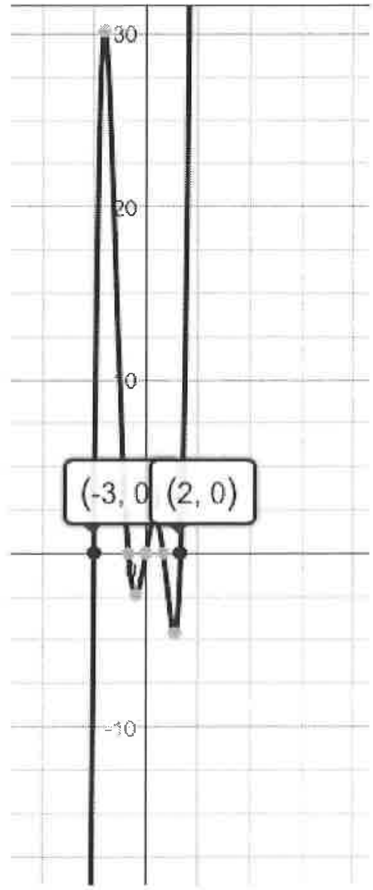
X-intercepts: (-3,0)(-1,0), (0,0), (1,0)

Factors: $x(x+3)(x+1)(x-1)$

Possible Equation:

$f(x) = x(x+3)(x+1)(x-1)$

F.)



End Behavior: $x \to \infty, f(x) \to$
$x \to -\infty, f(x) \to$

Leading Coefficient Sign: +

Degree of Function: 5

X-intercepts: (-3,0), (2,0), (0,0), (-1,0), (1,0)

Factors: $x(x+3)(x-2)(x+1)(x-1)$

Possible Equation:

$f(x) = x(x+3)(x-2)(x+1)(x-1)$

Reflection:

a) Why is finding the zeros for a polynomial important?

b) What do they tell us about that polynomial?

c) Name two ways to find the zeros.