Integrated Math 3
Unit 7: Modeling Rational Representations
7.1

Name: _____

Date: _____ Period: _____

Part I Objective: To simplify and multiply rational expressions

Warm Up:

1. Factor the following expressions.

a. $x^2 + 3x - 4$ $(x+4)(x-1)$

b. $4x^2 - 9$ $(2x+3)(2x-3)$

c. $6x^2 + x$ $x(6x+1)$

2. Simplify the following expressions.

a. $x^3x^2 =$ x^5

* add exponents when multiplying with the same base

b. $\frac{x^5}{x^2} =$ x^3

* subtract exponents when dividing with the same base

Vocabulary:

Simplified Form: A rational expression that has no common factors in the numerator and denominator.

Extraneous Solution: A solution of a simplified version of the equation that does not satisfy the original equation.

Excluded Value: The value of a variable that will make the denominator equal to zero.

Example 1: Simplify the expression and identify the excluded values.

a. $\frac{x^2+5x}{x^2} = \frac{x(x+5)}{x \cdot x}$

$= \frac{x+5}{x}$

Exclude: $x = 0$

$x^2 \neq 0$

$x \neq 0$

b. $\frac{6x+24}{x^2+7x+12} = \frac{6(x+4)}{(x+3)(x+4)}$

$= \frac{6}{x+3}$

Exclude: $x = -3, -4$

$(x+3)(x+4) \neq 0$

$x+3 \neq 0$ $\quad x+4 \neq 0$

$-3 \quad -3$ $\quad -4 \quad -4$

$x \neq -3$ $\quad x \neq -4$

Example 2: Multiply, simplify, and identify the excluded values.

a. $\frac{5x-15}{4x^2} \cdot \frac{x^3}{6x-18} = \frac{5(x-3)}{4x^2} \cdot \frac{x^3}{6(x-3)}$

$= \frac{5x}{4 \cdot 6} = \boxed{\frac{5x}{24}}$

Exclude: $x = 0, 3$

$\frac{4x^2}{4} \neq \frac{0}{4}$ $\quad \frac{6(x-3)}{6} \neq \frac{0}{6}$

$x^2 \neq 0$ $\quad x-3 \neq 0$

$x \neq 0$ $\quad +3 \; +3$

$x \neq 3$

b. $\frac{4xy^3}{x^2y} \cdot \frac{y}{8x} = \frac{4xy^{*3}}{8x^{3^2}y} = \frac{1y^3}{2x^2}$

$= \boxed{\frac{y^3}{2x^2}}$

Exclude: $x = 0, y = 0$

$8x^3y \neq 0$

c. $\frac{6x-12}{x^2-9x+18} \cdot \frac{7x-21}{5x-10}$

$= \frac{6(x-2)}{(x-6)(x-3)} \cdot \frac{7(x-3)}{5(x-2)}$

$= \boxed{\frac{42}{5(x-6)}}$

Exclude: $x = 2, 3, 6$

$(x-6)(x-3) \neq 0$ $\quad 5(x-2) \neq 0$

$x-6 \neq 0$ $\quad x-3 \neq 0$ $\quad \frac{5}{5} \quad \frac{5}{5}$

$+6 \; +6$ $\quad +3 \; +3$ $\quad x-2 \neq 0$

$x \neq 6$ $\quad x \neq 3$ $\quad +2 \; +2$

$x \neq 2$

d. $\frac{2x^2-2}{x^2-6x-7} \cdot \frac{(x^2-10x+21)}{1}$

$= \frac{2(x^2-1)}{(x-7)(x+1)} \cdot \frac{(x-7)(x-3)}{1}$

$= \frac{2(x+1)(x-1)}{(x-7)(x+1)} \cdot \frac{(x-7)(x-3)}{1}$

$= \boxed{2(x-1)(x-3)}$

Exclude: $x = -1, 7$

$(x-7)(x+1) \neq 0$ $\quad 1 \neq 0$ ✓

$x-7 \neq 0$ $\quad x+1 \neq 0$

$+7 \; +7$ $\quad -1 \; -1$

$x \neq 7$ $\quad x \neq -1$

Part II Objective: To divide rational expressions

Dividing Fractions

*****Dividing by a fraction is the same as multiplying by its reciprocal*****

Division Rule: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$

Example 3: Divide, simplify, and identify the excluded values.

a. $\frac{5x}{3x-12} \div \frac{x^2-2x}{x^2-6x+8} = \frac{5x}{3x-12} \cdot \frac{x^2-6x+8}{x^2-2x}$

$= \frac{5x}{3(x-4)} \cdot \frac{(x-2)(x-4)}{x(x-2)}$

$= \boxed{\frac{5}{3}}$

Exclude: $x = 0, 2, 4$

$\frac{3(x-4)}{3} \neq \frac{0}{3}$

$x-4 \neq 0$

$+4 \quad +4$

$x \neq 4$

$x(x-2) \neq 0$

$x \neq 0 \quad x-2 \neq 0$

$+2 \quad +2$

$x \neq 2$

b. $\frac{3}{x-8} \div \frac{x^2+3x}{x^2+x-6} = \frac{3}{x-8} \cdot \frac{x^2+x-6}{x^2+3x}$

$= \frac{3}{x-8} \cdot \frac{(x+3)(x-2)}{x(x+3)}$

$= \boxed{\frac{3(x-2)}{x(x-8)}}$

Exclude: $x = -3, 0, 8$

$x-8 \neq 0$

$+8 \quad +8$

$x \neq 8$

$x(x+3) \neq 0$

$x \neq 0 \quad x+3 \neq 0$

$-3 \quad -3$

$x \neq -3$

c. $\frac{x^2+5x-24}{2x+2} \div \frac{3x+24}{x^2-8x-9} = \frac{x^2+5x-24}{2x+2} \cdot \frac{x^2-8x-9}{3x+24}$

$= \frac{(x+8)(x-3)}{2(x+1)} \cdot \frac{(x+1)(x-9)}{3(x+8)}$

$= \boxed{\frac{(x-3)(x-9)}{6}}$

Exclude: $x = -8, -1$

$\frac{2(x+1)}{2} \neq \frac{0}{2}$

$x+1 \neq 0$

$-1 \quad -1$

$x \neq -1$

$\frac{3(x+8)}{3} \neq \frac{0}{3}$

$x+8 \neq 0$

$-8 \quad -8$

$x \neq -8$