

Objective: To add and subtract rational expressions with like and unlike denominators.

Warm Up: Find the sum or difference of the following fractions.

a) $\frac{1}{5} + \frac{2}{5} = \boxed{\frac{3}{5}}$

b) $\frac{1}{3} + \frac{2}{4} = \frac{4}{12} + \frac{6}{12} = \frac{10}{12} = \boxed{\frac{5}{6}}$

c) $\frac{15}{16} - \frac{3}{4} = \frac{15}{16} - \frac{12}{16} = \boxed{\frac{3}{16}}$

d) $\frac{18}{19} - \frac{2}{7} = \frac{126}{133} - \frac{38}{133} = \boxed{\frac{88}{133}}$

Fraction Rules:

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad \text{OR} \quad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

If the denominators are not the same, then you need to multiply the **individual pieces** by an expression to get the same denominator.

Example 1: Simplify the expression by adding or subtracting rational expressions with like denominators.

a. $\frac{7}{4x} + \frac{3}{4x} = \frac{10}{4x} = \boxed{\frac{5}{2x}}$

b. $\frac{2}{x+3} - \frac{4}{x+3} = \boxed{\frac{-2}{x+3}}$

c. $\frac{2x}{x+6} - \frac{5}{x+6} = \boxed{\frac{2x-5}{x+6}}$

Example 2: Simplify the expression by adding or subtracting rational expressions with unlike denominators.

$$3. \text{ a. } \frac{3}{4x^2} + \frac{2x}{12x} \cdot x$$

$$= \frac{9}{12x^2} + \frac{2x^2}{12x^2}$$

$$= \boxed{\frac{2x^2 + 9}{12x^2}}$$

$$\text{b. } \frac{5}{6x^2} + \frac{x}{4x^2 - 12x} = \frac{2(x-3) \cdot 5}{2(x-3) \cdot 6x^2} + \frac{x \cdot 3x}{4x(x-3) \cdot 3x}$$

$$= \frac{10(x-3)}{12x^2(x-3)} + \frac{3x^2}{12x^2(x-3)}$$

$$= \frac{3x^2 + 10(x-3)}{12x^2(x-3)}$$

$$= \boxed{\frac{3x^2 + 10x - 30}{12x^2(x-3)}}$$

$$\text{d. } \frac{4}{x^3} + \frac{x}{6x^3 + 3x^2} = \frac{3(2x+1) \cdot 4}{3(2x+1) \cdot x^3} + \frac{x \cdot x}{3x^2(2x+1) \cdot x} \quad \text{c. } \frac{4}{x^2} - \frac{8x-1}{2x^3}$$

$$= \frac{12(2x+1)}{3x^3(2x+1)} + \frac{x^2}{3x^3(2x+1)}$$

$$= \frac{x^2 + 12(2x+1)}{3x^3(2x+1)}$$

$$= \boxed{\frac{x^2 + 24x + 12}{3x^3(2x+1)}}$$

$$= \frac{8x}{2x^3} - \frac{8x-1}{2x^3}$$

$$= \frac{8x - (8x-1)}{2x^3}$$

$$= \frac{\cancel{8x} - \cancel{8x} + 1}{2x^3}$$

$$= \boxed{\frac{1}{2x^3}}$$

$$e. \frac{x+1}{x^2+4x+4} - \frac{2}{x^2-4} = \frac{\overset{(x-2)}{2} \cdot \overset{(x+2)}{x+1}}{\underset{(x+2)}{(x+2)(x+2)}} - \frac{\underset{(x+2)}{2} \cdot \overset{(x+2)}{(x+2)}}{\underset{(x+2)}{(x+2)(x-2)}} \quad f. \frac{x+1}{x^2+6x+9} - \frac{1}{x^2-9} = \frac{\overset{(x-3)}{(x-3)} \cdot \overset{(x+2)}{x+1}}{\underset{(x-3)}{(x+3)(x+3)}} - \frac{1 \cdot \overset{(x+3)}{(x+3)}}{\underset{(x-3)}{(x+3)(x-3)} \cdot \overset{(x+3)}{(x+3)}}$$

$$= \frac{(x-2)(x+1) - \overset{\text{red}}{2(x+2)}}{(x+2)(x+2)(x-2)}$$

$$= \frac{(x-2)(x+1) - 2x - 4}{(x+2)^2(x-2)}$$

	x	1
x	x ²	x
-2	-2x	-2

$$= \frac{x^2 - x - 2 - 2x - 4}{(x+2)^2(x-2)}$$

$$= \boxed{\frac{x^2 - 3x - 6}{(x+2)^2(x-2)}}$$

$$= \frac{(x-3)(x+1)}{(x+3)(x+3)(x-3)} - \frac{1(x+3)}{(x+3)(x+3)(x-3)}$$

$$= \frac{(x-3)(x+1) - \overset{\text{red}}{1(x+3)}}{(x+3)(x+3)(x-3)}$$

$$= \frac{(x-3)(x+1) - x - 3}{(x+3)^2(x-3)}$$

	x	1
x	x ²	x
-3	-3x	-3

$$= \frac{x^2 - 2x - 3 - x - 3}{(x+3)^2(x-3)}$$

$$= \boxed{\frac{x^2 - 3x - 6}{(x+3)^2(x-3)}}$$