

Objective: To solve rational equations.

Warm Up: Simplify the following expression: $\frac{7}{x-3} + \frac{2x}{x-6} \cdot \frac{(x-3)}{(x-3)}$

$$= \frac{7(x-6) + 2x(x-3)}{(x-6)(x-3)} = \frac{7x - 42 + 2x^2 - 6x}{(x-6)(x-3)} = \frac{2x^2 + x - 42}{(x-6)(x-3)}$$

Example 1: Solve. Be sure to check for excluded values!

a. $\frac{3}{x^2+4x} = \frac{1}{x+4} \Rightarrow \frac{3}{x(x+4)} = \frac{1}{x+4} \cdot x$

$$\frac{3}{x(x+4)} = \frac{x}{x(x+4)}$$

$x = 3$

Exclude: $x = -4, 0$

$$x(x+4) \neq 0$$

$$\downarrow \quad \downarrow$$

$$x \neq 0 \quad x+4 \neq 0$$

$$\quad \quad \quad -4 \quad -4$$

$$\quad \quad \quad \underline{\quad \quad}$$

$$\quad \quad \quad x \neq -4$$

b. $\frac{6}{2x^2+2x} = \frac{x-2}{x+1}$

$$\Rightarrow \frac{6}{2x(x+1)} = \frac{x-2}{x+1} \cdot \frac{2x}{2x}$$

$$\frac{6}{2x(x+1)} = \frac{2x(x-2)}{2x(x+1)}$$

$$6 = 2x(x-2)$$

$$6 = 2x^2 - 4x$$

$$0 = 2x^2 - 4x - 6$$

$$0 = 2(x^2 - 2x - 3)$$

$$0 = \frac{2}{2}(x+1)(x-3)$$

$$0 = (x+1)(x-3) \rightarrow x = -1, 3$$

Exclude: $x = -1, 0$

$$2x(x+1) \neq 0$$

$$\downarrow \quad \downarrow$$

$$\frac{2x}{2} \neq 0 \quad x+1 \neq 0$$

$$x \neq 0 \quad \underline{-1 \quad -1}$$

$$\quad \quad \quad x \neq -1$$

$$c. \frac{x^2}{x^2-x} = \frac{1}{x-1} \Rightarrow \frac{x^2}{x(x-1)} = \frac{1}{x-1} \cdot x$$

$$\frac{x^2}{x(x-1)} = \frac{x}{x(x-1)}$$

$$x^2 = x$$

$$x^2 - x = 0$$

$$x(x-1) = 0$$

$$x = \emptyset, 1$$

No Solution

Exclude: $x = 0, 1$

$$x(x-1) \neq 0$$

$$\downarrow \quad \downarrow$$

$$x \neq 0 \quad x-1 \neq 0$$

$$\quad \quad \quad +1 \quad +1$$

$$\quad \quad \quad \hline \quad \quad \quad x \neq 1$$

Example 2: Solve each of the following. Be sure to check for excluded values.

$$a. \frac{3}{x+4} - \frac{2x}{x+4} = \frac{5x}{x+4} \Rightarrow \frac{3-2x}{x+4} = \frac{5x}{x+4}$$

Exclude: $x = -4$

$$3-2x = 5x$$

$$3 = 7x$$

$$x = \frac{3}{7}$$

$$x+4 \neq 0$$

$$\quad -4 \quad -4$$

$$\quad \quad \quad \hline \quad \quad \quad x \neq -4$$

$$b. \frac{8}{x-1} + \frac{6}{x+1} = \frac{12}{(x+1)(x-1)} \Rightarrow \frac{8(x+1) + 6(x-1)}{(x+1)(x-1)} = \frac{12}{(x+1)(x-1)}$$

Exclude: $x = -1, 1$

$$8x+8 + 6x-6 = 12$$

$$14x + 2 = 12$$

$$14x = 10$$

$$x = \frac{10}{14}$$

$$x = \frac{5}{7}$$

$$(x-1)(x+1) \neq 0$$

$$\downarrow \quad \quad \downarrow$$

$$x-1 \neq 0 \quad x+1 \neq 0$$

$$\quad +1 \quad +1 \quad \quad \quad -1 \quad -1$$

$$\quad \quad \quad \hline \quad \quad \quad x \neq 1 \quad \quad \quad x \neq -1$$

$$c. \frac{10}{x^2-2x} + \frac{4}{x} = \frac{5}{x-2} \Rightarrow \frac{10}{x(x-2)} + \frac{4^{(x-2)}}{x(x-2)} = \frac{5 \cdot x}{x-2 \cdot x}$$

$$\frac{10+4(x-2)}{\cancel{x(x-2)}} = \frac{5x}{\cancel{x(x-2)}}$$

$$10+4x-8=5x$$

$$4x+2=5x$$

$$x=2$$

No Solution

Exclude: $x=0,2$

$$\begin{array}{l} x(x-2) \neq 0 \\ \downarrow \qquad \downarrow \\ x \neq 0 \qquad x-2 \neq 0 \\ \qquad \qquad \quad +2 \quad +2 \\ \hline \qquad \qquad \quad x \neq 2 \end{array}$$