Integrated Math 3
Unit 7: Modeling Rational Representations
7.7

Name: ________________

Date: __________ Period: ______

Objective: To determine key features of a rational function including domain, range, intercepts, and asymptotes.

Key Terms:

- **Rational function**: A function in the form $y = \frac{p(x)}{q(x)}$ where the denominator **cannot equal zero** (excluded values!!).
- **Hyperbola**: The "U" shape of a graph that a rational function takes.
- **Branches**: Two symmetrical points on a graph
- **Asymptote**: A line where the graph will not cross
  - **Vertical Asymptote**: A line located at the value of the value of the variable that makes the value of the denominator equal zero
  - **Horizontal Asymptote**: A line at the value $y = \frac{a}{c}$ or $y = k$ depending on the type of equation you have

Part I: Determining Horizontal Asymptotes:

Example 1: The " $y = \frac{ax^m}{bx^n}$ " form.

When there are variables on both the numerator and the denominator, there are three options for the existence of a horizontal asymptote:

1. If $m = n$, then $y = \frac{a}{b}$
2. If $m < n$, then $y = 0$
3. If $m > n$, then there is no horizontal asymptote

*look for the highest power

A.) $y = \frac{2x+4}{6x-2}$ $y = \frac{2x}{6x}$ $y = \frac{1}{3}$	B.) $y = \frac{x-1}{x^2-4}$ $y = \frac{0x^2}{1x^2}$ $y = 0$	C.) $y = \frac{x^3-1}{2x-4}$ $y = \frac{1x^3}{0x^3} \Rightarrow$ Undefined No horizontal asymptote

Example 2: The " $y = \frac{a}{x-h} + k$ " form:

When there are only variables on the denominator, the horizontal asymptote exists at the line $y = k$

A.) $y = \frac{2}{x-1} + 4$ $y = 4$	B.) $y = \frac{6}{x+2} - 3$ $y = -3$

Part II: Determining Vertical Asymptotes

The values that make the function undefined.

The values of x that make the denominator of a rational function equal zero and are unique only to the denominator indicate the location of any vertical asymptotes.

Essentially, the vertical asymptotes are excluded values that are left over after everything in the function is simplified/canceled out.

Example 1:

A.) $y = \frac{5x+15}{2x-2} = \frac{5(x+3)}{2(x-1)}$

$\frac{2(x-1)}{2} \neq \frac{0}{2}$

$x-1 \neq 0$

$+1 \quad +1$

$x \neq 1$

$\boxed{x=1}$

B.) $y = \frac{4x+16}{x^2+9x+20} = \frac{4\cancel{(x+4)}}{\cancel{(x+4)}(x+5)} = \frac{4}{x+5}$

$x+5 \neq 0$

$-5 \quad -5$

$x \neq -5$

$\boxed{x=-5}$

Part III: Determining the x-intercepts

The values that make the function zero.

The values of x that make the numerator of a rational function equal zero.

A.) $y = \frac{5x+15}{2x-2} = \frac{5(x+3)}{2(x-1)}$

$\frac{5(x+3)}{5} = \frac{0}{5}$

$x+3=0$

$-3 \quad -3$

$x=-3$

$\boxed{(-3,0)}$

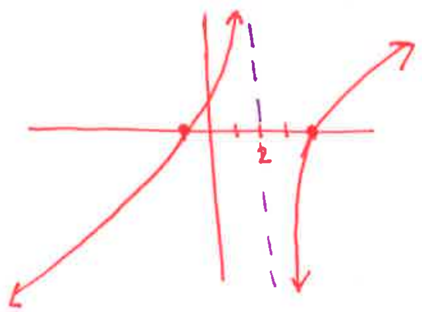
B.) $y = \frac{4x+16}{x^2+9x+20} = \frac{4\cancel{(x+4)}}{\cancel{(x+4)}(x+5)} = \frac{4}{x+5}$

$4 \neq 0$

$\boxed{\text{No x-intercepts}}$

Part IV: Putting it all together

$$y = \frac{x^2-3x-4}{x-2} = \frac{(x-4)(x+1)}{x-2}$$



Horizontal Asymptote: $y = \frac{1x^2}{0x^2} \Rightarrow$ Und. None

Vertical Asymptote(s): $x=2$

X-intercept(s): $(4,0), (-1,0)$

Domain: $(-\infty, 2)\ (2, \infty)$

Range: $(-\infty, \infty)$

$$\frac{-2-1(x+3)}{x+3} = \frac{-2-x-3}{x+3}$$

$$= \frac{-x-5}{x+3} = \boxed{\frac{-(x+5)}{x+3}}$$

Let's Practice!

A.) $y = \frac{x+1}{2x-4} = \frac{x+1}{2(x-2)}$

Horizontal Asymptote: $y = \frac{1x}{2x}$ $y = \frac{1}{2}$

Vertical Asymptote(s): $x = 2$

X-intercept(s): $(-1, 0)$

Domain: $(-\infty, 2)\ (2, \infty)$

Range: $(-\infty, \frac{1}{2})\ (\frac{1}{2}, \infty)$

B.) $y = -\frac{2}{x+3} - 1\frac{(x+3)}{1(x+3)}$

Horizontal Asymptote: $y = -1$

Vertical Asymptote(s): $x = -3$

X-intercept(s): $(-5, 0)$

Domain: $(-\infty, -3)\ (-3, \infty)$

Range: $(-\infty, -1)\ (-1, \infty)$

C.) $y = \frac{3}{x-1} + 2\frac{(x-1)}{1(x-1)}$ $\frac{3+2(x-1)}{x-1} = \frac{3+2x-2}{x-1} = \boxed{\frac{2x+1}{x-1}}$

Horizontal Asymptote: $y = 2$

Vertical Asymptote(s): $x = 1$

X-intercept(s): $(-\frac{1}{2}, 0)$

Domain: $(-\infty, 1)\ (1, \infty)$

Range: $(-\infty, 2)\ (2, \infty)$

D.) $y = \frac{3x^2}{x^2-4} = \frac{3x^2}{(x+2)(x-2)}$

Horizontal Asymptote: $y = \frac{3x^2}{1x^2} \Rightarrow y = 3$

Vertical Asymptote(s): $x = -2,\ x = 2$

X-intercept(s): $(0, 0)$

Domain: $(-\infty, -2)\ (-2, 2)\ (2, \infty)$

Range: $(-\infty, 3)\ (3, \infty)$

E.) $y = \frac{x^2-2x-3}{x+4} = \frac{(x-3)(x+1)}{x+4}$

Horizontal Asymptote: $y = \frac{1x^2}{0x^2}$ None

Vertical Asymptote(s): $x = -4$

X-intercept(s): $(3, 0), (-1, 0)$

Domain: $(-\infty, -4)\ (-4, \infty)$

Range: $(-\infty, \infty)$

F.) $y = \frac{x}{x^2+1}$

Horizontal Asymptote: $y = \frac{0x^2}{1x^2} \Rightarrow y = 0$

Vertical Asymptote(s): None

X-intercept(s): $\cancel{(0, 0)}$ None (due to the horizontal asymptote)

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0)\ (0, \infty)$

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7.8

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Date:__________ Period:______

Objective: To determine key features of a rational function including domain, range, intercepts, and asymptotes from a graph.

Warm Up:

From the following equation, $y = \frac{3x-18}{x^2-10x+24}$, determine the following key features: $\frac{3\cancel{(x-6)}}{\cancel{(x-6)}(x-4)}$

Horizontal asymptote: $y=\frac{0x^2}{1x^2} \Rightarrow y=0$

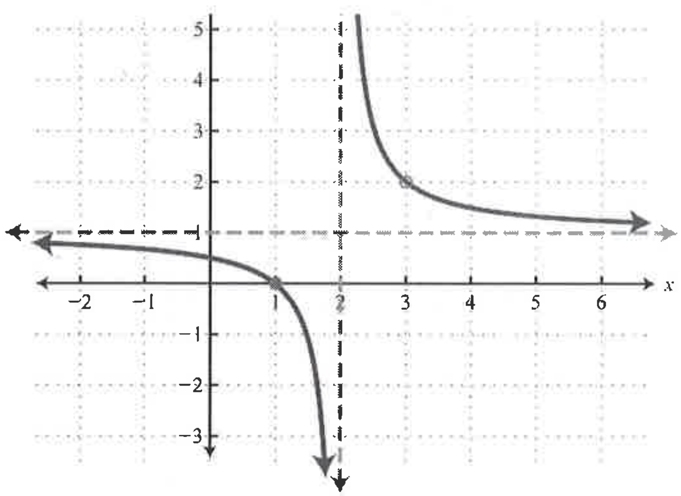
Vertical asymptote(s): $x=4$

X-intercept(s): None

Domain: $(-\infty,4)\ (4,\infty)$

Range: $(-\infty,0)\ (0,\infty)$

Example 1: Determine the key features from the graphs below.



Horizontal asymptote: $y=1$

Vertical asymptote(s): $x=2$

X-intercept(s): $(1,0)$

Domain: $(-\infty,2)\ (2,\infty)$

Range: $(-\infty,1)(1,\infty)$

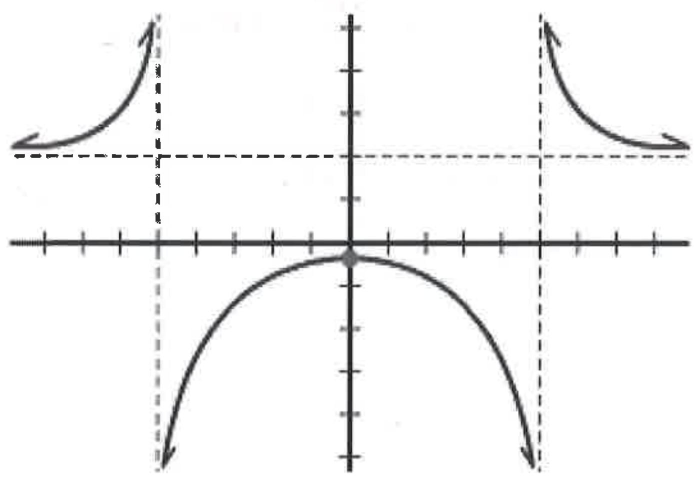
Horizontal asymptote: $y=2$

Vertical asymptote(s): $x=-5,\ x=5$

X-intercept(s): None

Domain: $(-\infty,-5)\ (-5,5)\ (5,\infty)$

Range: $(-\infty,-\frac{1}{3})\ (2,\infty)$



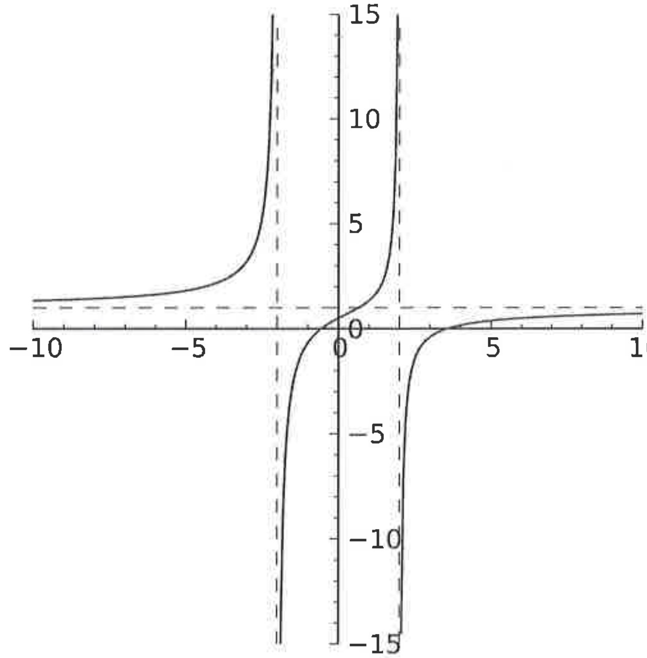
Horizontal asymptote: $y=1$

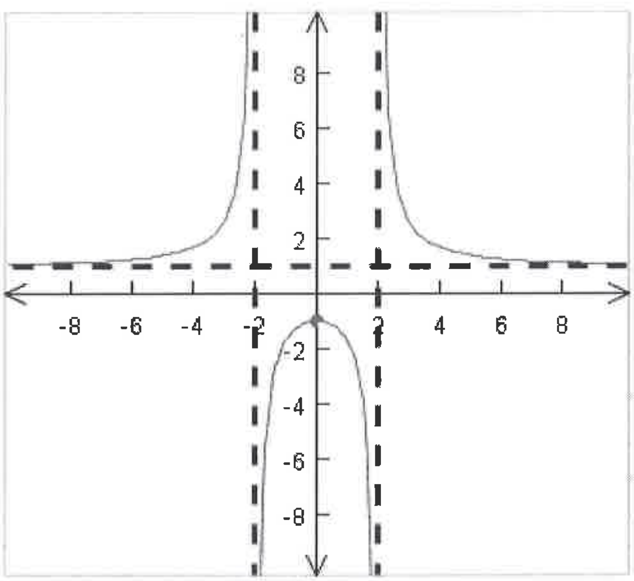
Vertical asymptote(s): $x=-2,\ x=2$

X-intercept(s): $(3.5, 0)$

Domain: $(-\infty,-2)\ (-2,2)\ (2,\infty)$

Range: $(-\infty,1)\ (1,\infty)$





Horizontal asymptote: $y=1$

Vertical asymptote(s): $x=-2,\ x=2$

X-intercept(s): None

Domain: $(-\infty,-2)\ (-2,2)\ (2,\infty)$

Range: $(-\infty,-1)\ (1,\infty)$