

**Objective:** To determine the inverse of a rational function.

**Recall:**

An **inverse** function has the unique property of mirroring a function over the line  $y = x$ .

How to Determine an Inverse Function

- Change the function notation to  $f(x) =$   $y =$
- Switch  $x$  and  $y$
- Solve for  $y$  (get  $y$  alone / by itself)
- Rewrite  $y$  using function notation ( $f^{-1}(x) =$ )

**Example 1:**  $f(x) = \frac{3}{x+2}$

$$y = \frac{3}{x+2}$$

$$(y+2)x = \frac{3}{y+2} (y+2)$$

$$x(y+2) = 3$$

$$\begin{array}{r} xy + 2x = 3 \\ -2x \quad -2x \\ \hline \end{array}$$

$$\frac{xy}{x} = \frac{3-2x}{x}$$

$$y = \frac{3-2x}{x}$$

$$f^{-1}(x) = \frac{3-2x}{x}$$

or

$$f^{-1}(x) = \frac{3}{x} - 2$$

**Example 2:**  $g(x) = \frac{x+4}{x}$

$$y = \frac{x+4}{x}$$

$$y \cdot x = \frac{y+4}{y} \cdot y$$

$$f^{-1}(x) = \frac{4}{x-1}$$

$$xy = y+4$$

$$\begin{array}{r} -y \quad -y \\ \hline \end{array}$$

$$xy - y = 4$$

$$\frac{y(x-1)}{x-1} = \frac{4}{x-1}$$

} factor out  
the GCF

$$y = \frac{4}{x-1}$$

**Example 3:**  $h(x) = \frac{2x-5}{x+3}$

$$y = \frac{2x-5}{x+3}$$

$$(y+3) \cdot x = \frac{2y-5}{y+3} \cdot (y+3)$$

$$x(y+3) = 2y-5$$

$$xy + 3x = 2y - 5$$

$$\begin{array}{r} -2y \quad -2y \\ \hline \end{array}$$

$$xy - 2y + 3x = -5$$

$$\begin{array}{r} -3x \quad -3x \\ \hline \end{array}$$

$$xy - 2y = -3x - 5$$

$$\frac{y(x-2)}{x-2} = \frac{-3x-5}{x-2}$$

} factor out  
the GCF

$$y = \frac{-3x-5}{x-2}$$

$$f^{-1}(x) = \frac{-3x-5}{x-2}$$