Integrated Math 3
Unit 8: Exponential & Logarithmic Functions
8.2

Name: ______________________

Date:__________ Period:______

Objective: To solve equations with uncommon bases and to convert between different forms

Warm Up

Simplify: $27^{x+1} * \left(\frac{1}{3}\right)^{x-2} * 9^x$

$= (3^3)^{x+1} \cdot (3^{-1})^{x-2} \cdot (3^2)^x$

$= 3^{3x+3} \cdot 3^{-1x+2} \cdot 3^{2x}$

$= 3^{3x+3+-1x+2+2x}$

$= \boxed{3^{4x+5}}$

Solve: $4^{x+1} = 16^{2x-4}$

$4^{x+1} = (4^2)^{2x-4}$

$4^{x+1} = 4^{4x-8}$

$x+1 = 4x-8$

$9 = 3x$

$\boxed{x=3}$

Simplifying and solving equations provide nice neat solutions in the questions above because the expressions can be written with a common base. If two random numbers are chosen, the chances are relatively small that they share a common base, though.

Let's Investigate!

Estimate the value of the variable to the nearest hundredth.

A.) $2^x = 5$

B.) $3^a = 25$

C.) $5^y = 15$

D.) $4^b = 3$

E.) $10^y = 500$

F.) $2 \cdot 3^x - 4 = 5$

$\quad +4 \quad +4$

$\frac{2 \cdot 3^x}{2} = \frac{9}{2}$

$3^x = \frac{9}{2}$

The **logarithm** is the inverse operation to exponentiation. That means the **logarithm** of a number is the exponent to which another fixed value, the base, must be raised to produce that number.

$$\boxed{b^x = a} \quad \text{is equivalent to:} \quad \boxed{\log_b a = x}$$

"a base to an exponent gives you an answer" "log with a base of an answer gives you an exponent"

This will provide you with a more efficient way of solving for the variables!!!

Example 1:
Rewrite each equation below in logarithmic notation.

A.) $2^x = 5$

$\log_2 5 = x$

B.) $3^a = 25$

$\log_3 25 = a$

C.) $5^y = 15$

$\log_5 15 = y$

D.) $4^b = 3$

$\log_4 3 = b$

E.) $10^y = 500$

When the base is ten, we don't need to write it.

$\log_{10} 500 = y$

$\Rightarrow \boxed{\log 500 = y}$

F.) $2 \cdot 3^x - 4 = 5$

$+4 \quad +4$

$\frac{2 \cdot 3^x}{2} = \frac{9}{2}$

$3^x = \frac{9}{2} \rightarrow \boxed{\log_3 \frac{9}{2} = x}$

In order to solve for the variables, we will use something called the "change of base" formula:

Change of Base Formula:

(How to solve for an exponent when you do not common bases)

If given: $b^x = a$

Then: $\log_b a = x$

Which implies: $x = \dfrac{\log a}{\log b}$

Example 2:
Find the exact value for the variable to the nearest hundredth.

A.) $2^x = 5$

$\log_2 5 = x$

$x = \dfrac{\log 5}{\log 2}$

$\boxed{x \approx 2.32}$

B.) $3^a = 25$

$\log_3 25 = a$

$a = \dfrac{\log 25}{\log 3}$

$\boxed{a \approx 2.93}$

C.) $5^y = 15$

$\log_5 15 = y$

$y = \dfrac{\log 15}{\log 5}$

$\boxed{y \approx 1.68}$

D.) $4^b = 3$

$\log_4 3 = b$

$b = \dfrac{\log 3}{\log 4}$

$\boxed{b \approx .79}$

E.) $10^y = 500$

$\log 500 = y$

$\boxed{y \approx 2.70}$

F.) $2 \cdot 3^x - 4 = 5$

$\log_3 \frac{9}{2} = x$

$x = \dfrac{\log \frac{9}{2}}{\log 3}$

$\boxed{x \approx 1.37}$

Let's Practice!

Example 3:
Complete the table by switching back & forth between exponential form and logarithmic form.

Exponential Form	**Logarithmic Form**
$4^2 = 16$	$\log_4 16 = 2$
$1^{99} = 1$	$\log_1 1 = 99$
$2^8 = 256$	$\log_2 256 = 8$
$3^0 = 1$	$\log_3 1 = 0$
$10^2 = 100$	$\log 100 = 2$
$a^b = c$	$\log_a c = b$

assume base of 10

Example 4:
Rewrite the logarithmic form to exponential form & then evaluate/solve each one.

A.) $\log_3 9 = x$

$3^x = 9$

$\boxed{x = 2}$

B.) $\log_2 \frac{1}{16} = x$

$2^x = \frac{1}{16}$

$\boxed{x = -4}$

C.) $\log_4 1 = x$

$4^x = 1$

$\boxed{x = 0}$

D.) $\log_2 x = -3$

$2^{-3} = x$

$\boxed{x = \frac{1}{8}}$

E.) $\log_x 16 = 2$

$x^2 = 16$

$\boxed{x = 4}$

F.) $\log_x \frac{1}{9} = -2$

$x^{-2} = \frac{1}{9}$

$\boxed{x = 3}$