

Objective: To evaluate and rewrite functions using the natural logarithm.

Warm Up: Evaluate.

A.) $\log_{10} 100 = x$

$$10^x = 100$$

$$x = 2$$

B.) $\log_{10} 10 = x$

$$10^x = 10$$

$$x = 1$$

When working with logarithms, there were many instances when we did not see a subscript beside the abbreviation "log" to indicate the base. We assume when a base is not present, the logarithm has base of 10 by default.

Key Term:

Natural logarithm (natural log) is a specific type of logarithm that has a different base, e .

The natural log is abbreviated: \ln and it works identically to \log s.

$$\boxed{e^x = a} \text{ is equivalent to } \boxed{\log_e a = x} \text{ which is the same as } \boxed{\ln a = x}$$

Example 1: Rewrite the following in logarithmic form

A.) $e^0 = 1$

$$\log_e 1 = 0$$

$$\Rightarrow \ln 1 = 0$$

B.) $e^1 = e$

$$\log_e e = 1$$

$$\Rightarrow \ln e = 1$$

C.) $e^{10} = 27.18$

$$\log_e 27.18 = 10$$

$$\Rightarrow \ln 27.18 = 10$$

Example 2: Rewrite the following in exponential form

A.) $\ln 6 = x$

$$e^x = 6$$

B.) $\ln e = 1$

$$e^1 = e$$

C.) $\ln 148.41 = 5$

$$e^5 = 148.41$$

Example 3: Evaluate. If necessary, round to the nearest tenth.

$$\begin{aligned} \text{A.) } e^4 e^7 &= e^{4+7} \\ &= e^{11} \\ &\approx \boxed{59874.1} \end{aligned}$$

$$\begin{aligned} \text{B.) } e^{-3} e^7 &= e^{-3+7} \\ &= e^4 \\ &\approx \boxed{54.6} \end{aligned}$$

$$\begin{aligned} \text{C.) } e^9 + e^0 &= e^9 + 1 \\ &\approx 8103.1 + 1 \\ &\approx \boxed{8104.1} \end{aligned}$$

$$\begin{aligned} \text{D.) Solve } \frac{4 \ln x}{4} &= \frac{23}{4} \\ \ln x &= \frac{23}{4} \\ e^{23/4} &= x \\ &\approx \boxed{314.2} \end{aligned}$$

$$\begin{aligned} \text{E.) Solve } \frac{5 \ln 3x}{5} &= \frac{14}{5} \\ \ln 3x &= \frac{14}{5} \\ e^{14/5} &= 3x \\ \frac{16.4}{3} &= \frac{3x}{3} \\ &\approx \boxed{x \approx 3.5} \end{aligned}$$

$$\begin{aligned} \text{F.) Solve } \frac{6(e^{x+1})}{6} &= \frac{1,000,000}{6} \\ e^{x+1} &= 166666.7 \\ \ln(166666.7) &= x+1 \\ \ln(166666.7) - 1 &= x \\ &\approx \boxed{x \approx 11.0} \end{aligned}$$

$$\begin{aligned} \text{G.) Solve } \frac{-3(e^{2x-3})}{-3} &= \frac{-469.016}{-3} \\ e^{2x-3} &= 156.3 \\ \ln 156.3 &= 2x-3 \\ \ln 156.3 + 3 &= 2x \\ \frac{\ln 156.3 + 3}{2} &= x \\ &\approx \boxed{x \approx 4.0} \end{aligned}$$