Integrated Math III **Name:** _____

Per: _____ **Date:** _____

Unit 6 Test Study Guide
Take note of NON-CALCULATOR questions

1. Write each polynomial in standard form and classify by degree and number of terms: **(NON-CALC)**

a. $x^2 - 4$

Quadratic
Binomial

b. $x^3 - 4x^3 + 6x^3 \Rightarrow 3x^3$

Cubic
Monomial

c. $x^2 - x^3 - x^4 + 5x^3 \Rightarrow -x^4 + 4x^3 + x^2$

Quartic
Trinomial

d. 172

Constant
Monomial

2. Solve the following polynomial equations.

c. $(x-3)(x^2-1) = 0$

$\Rightarrow (x-3)(x-1)(x+1) = 0$

$x-3=0$ $\quad x-1=0$ $\quad x+1=0$

$+3\ +3$ $\quad +1\ +1$ $\quad -1\ -1$

$x=3$ $\quad x=1$ $\quad x=-1$

$(3,0), (1,0), (-1,0)$

d. $(x^2-25)(x^2+5x+4) = 0$

$\Rightarrow (x-5)(x+5)(x+4)(x+1) = 0$

$x-5=0$ $\quad x+5=0$ $\quad x+4=0$ $\quad x+1=0$

$+5\ +5$ $\quad -5\ -5$ $\quad -4\ -4$ $\quad -1\ -1$

$x=5$ $\quad x=-5$ $\quad x=-4$ $\quad x=-1$

	x	4
x	x^2	$4x$
1	$1x$	4

Mult: $4x^2$
Add: $5x$

$(5,0), (-5,0), (-4,0), (-1,0)$

e. $10x^3 + 5x^2 - 2x - 1 = 0$

	$2x$	1
$5x^2$	$10x^3$	$5x^2$
-1	$-2x$	-1

$(5x^2-1)(2x+1) = 0$

$5x^2-1=0$ $\quad 2x+1=0$

$+1\ +1$ $\quad -1\ -1$

$\frac{5x^2}{5} = \frac{1}{5}$ $\quad \frac{2x}{2} = \frac{-1}{2}$

$x^2 = 1/5$ $\quad x = -1/2$

$x = \pm\sqrt{1/5}$

$(\sqrt{\frac{1}{5}}, 0), (-\sqrt{\frac{1}{5}}, 0), (-\frac{1}{2}, 0)$

f. $\sqrt{x+7} = (x-1)^2$ (hint: use your calculator!)

$2^{nd} \rightarrow$ trace (calc) $\rightarrow$ 5: intersect

$Y_1 = \sqrt{x+7}$

$Y_2 = (x-1)^2$

$x = -0.591, 2.768$

$(-0.591, 0), (2.768, 0)$

g. $5x^2 = 20x$ *must equal zero

$-20x \quad -20x$

$5x^2 - 20x = 0$

$5x(x-4) = 0$

$\frac{5x}{5} = \frac{0}{5}$ $\quad x-4=0$

$+4\ +4$

$x=0$ $\quad x=4$

$(0,0), (4,0)$

h. $4x^2 - 15x - 25 = 0$

	x	-5
$4x$	$4x^2$	$-20x$
5	$5x$	-25

Mult: $-100x^2$
Add: $-15x$

$(4x+5)(x-5) = 0$

$4x+5=0$ $\quad x-5=0$

$-5\ -5$ $\quad +5\ +5$

$\frac{4x}{4} = \frac{-5}{4}$ $\quad x=5$

$x = -5/4$

$(-5/4, 0), (5, 0)$

3. Is $(4x + 3)$ a factor of $(12x^3 - 11x^2 + 9x + 18)$?

$4x+3=0$
$-3\ -3$
$\frac{4x}{4}=\frac{-3}{4}$
$x=-3/4$

$12(-\frac{3}{4})^3-11(-\frac{3}{4})^2+9(-\frac{3}{4})+18$
$= 0$

Yes, it is a factor!

4. Is $x = -1$ a solution of $(3x^3 + 5x - 1)$?

$3(-1)^3+5(-1)-1$
$=-9$

No! It's not a solution.

5. The polynomial $g(x) = x^3 - 9x^2 + 26x - 24$ has values in the table provided below.

x	-2	-1	0	1	2
$g(x)$	-120	-60	-24	-6	0

a. What is the x-intercept?

(2,0) → $(x-2)$

b. Use the table to help you write the polynomial so it is in factored form.
(Hint: use the x-intercept and an area model to divide)

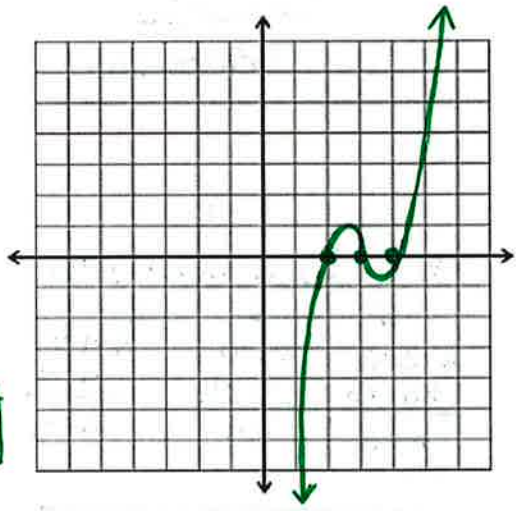
$(x^3-9x^2+26x-24)\div(x-2)$

	x^2	$-7x$	12
x	x^3	$-7x^2$	$12x$
-2	$-2x^2$	$14x$	-24

$\Rightarrow g(x)=(x-2)(x^2-7x+12)$

	x	-3
x	x^2	$-3x$
-4	$-4x$	12

Mult: $12x^2$
Add: $-7x$



Factored Form: $g(x) =$ $(x-2)(x-4)(x-3)$

c. Graph the polynomial.

6. Use the <u>area model</u> to divide and solve the problem below.

a. If the polynomial $h(x) = -x^3 - 16x^2 - 75x - 108$ has the x-intercept $x = -4$. Find all other x-intercepts of the polynomial. Write the polynomial in factored form and graph it.

→ $(x+4)$

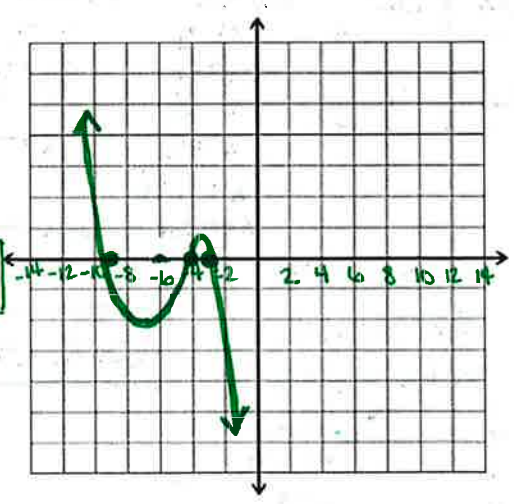
	$-x^2$	$-12x$	-27
x	$-x^3$	$-12x^2$	$-27x$
4	$-4x^2$	$-48x$	-108

$h(x)=(x+4)(-x^2-12x-27)$
GCF: -1
$h(x)=-(x^3+12x+27)(x+4)$

$h(x)=-(x+3)(x+9)(x+4)$

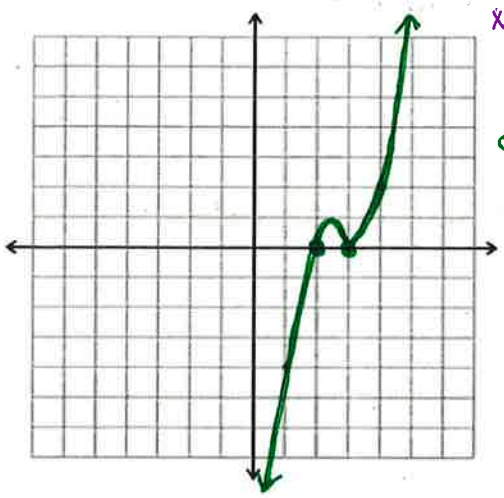
	x	9
x	x^2	$9x$
3	$3x$	27

Mult: $27x^2$
Add: $12x$



7. Graph the following polynomials. Be sure to consider multiplicity (repeating x-intercepts) **(NON-CALC)**

a. $f(x) = (x-2)(x-3)^2$

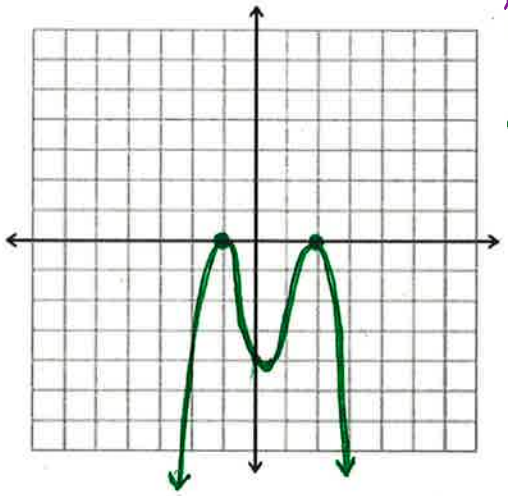


x-int: (2,0), (3,0)*

odd degree (3)

+

b. $g(x) = -(x+1)^2(x-2)^2$



x-int: (-1,0)*, (2,0)*

even degree (4)

−

8. Sketch the end behavior of the following polynomial functions. **(NON-CALC)**

a. $x^5 - x^7$ degree: 7 (odd)

⇒ $-x^7+5$ sign: −

b. $2x - 5 + x + 2 - x^3$ degree: 3 (odd)

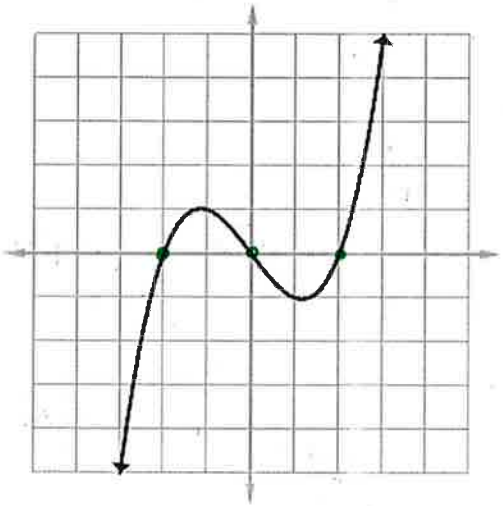
⇒ $-x^3+3x-3$

c. $x^4 - 7 + x$ degree: 4 (even)

sign: − ⇒ x^4+x-7 sign: +

9. Given the following graphs, determine the information below. **(NON-CALC)**

a.



End Behavior: $x \to \infty, f(x) \to \infty$

$x \to -\infty, f(x) \to -\infty$

Leading Coefficient Sign: +

Degree of Function: odd 3 (2 turns)

x-intercepts: (-2,0), (0,0), (2,0)

Factors: x (x+2)(x-2)

Equation in Factored Form: $f(x) = x(x+2)(x-2)$ or $f(x) = x(x^2-4)$

Y-intercept: (0,0)

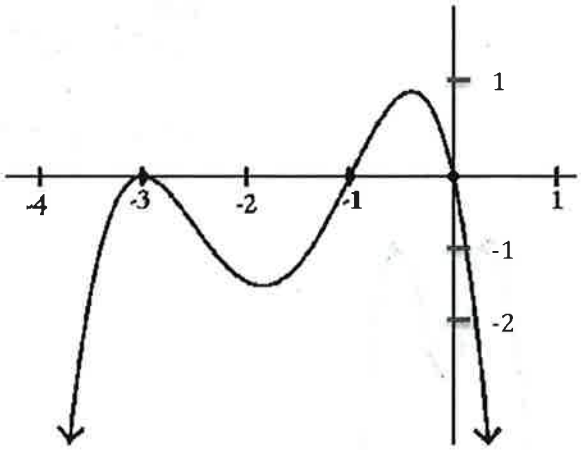
Relative Minimum(s): (1,-1)

Relative Maximum(s): (-1,1)

Increasing Intervals: $(-\infty,-1)$ $(1,\infty)$

Decreasing Intervals: (-1,1)

b. **(NON-CALC)**



End Behavior: $x \to \infty,\ f(x) \to -\infty$

$x \to -\infty,\ f(x) \to -\infty$

Leading Coefficient Sign: $-$

Degree of Function: even (4)

→ 3 turns

X-intercepts: $(-3,0)^*, (-1,0), (0,0)$

Factors: $x(x+3)(x+1)$

Equation in Factored Form:

$f(x) = -x(x+3)^2(x+1)$

Y-intercept: $(0,0)$

Relative Minimum(s): $(-1.8,-1.5)$

Relative Maximum(s): $(-3,0), (-0.4,1)$

Increasing Intervals: $(-\infty,-3)\ (-1.8,-0.4)$

Decreasing Intervals: $(-3,-1.8)\ (-0.4,\infty)$

10. Given the following functions, solve for their inverses and then verify the results graphically.

a. $h(x) = 3(x-1)$

$y = 3(x-1)$

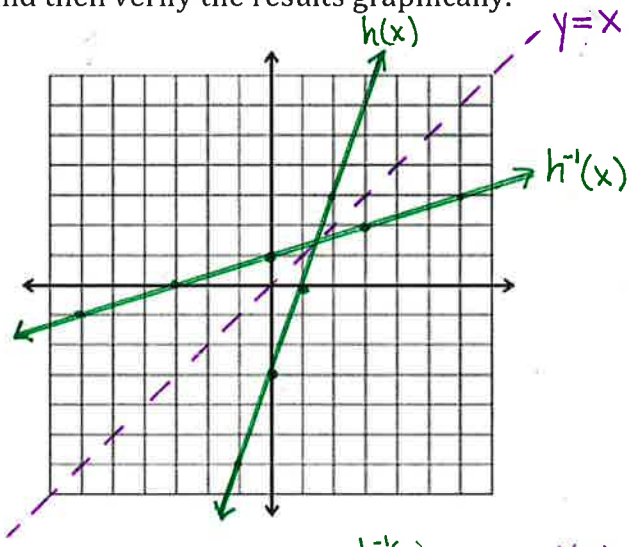
$\frac{x}{3} = \frac{3(y-1)}{3}$

$\frac{x}{3} = y - 1$

$+1 \quad +1$

$\frac{x}{3} + 1 = y$

$h^{-1}(x) = \frac{x}{3} + 1$



b. $h(x) = \frac{1}{5}(x+6)$

$y = \frac{1}{5}(x+6)$

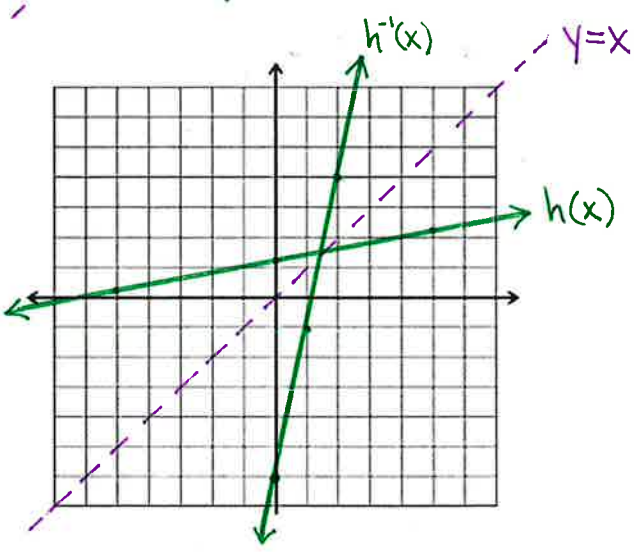
$5 \cdot x = \frac{1}{5}(y+6) \cdot 5$

$5x = y + 6$

$-6 \quad -6$

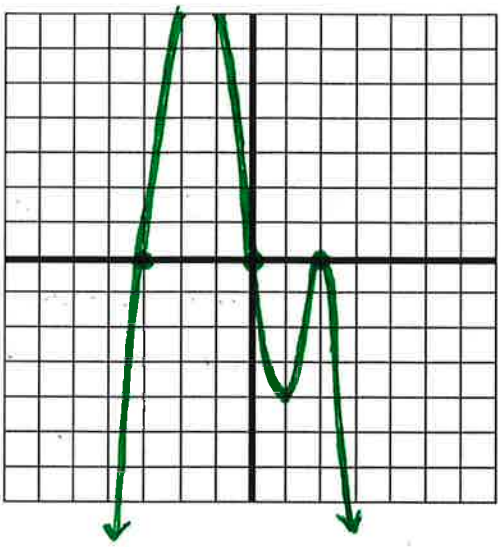
$5x - 6 = y$

$h^{-1}(x) = 5x - 6$



11. Graph each of the polynomial below.

$$f(x) = -x(x-2)^2(x+3)$$



Classify *(by degree):*

Degree: 4 (Even) / Odd; LC: − + / −

End Behavior: $x \to \infty,\ f(x) \to -\infty$

$x \to -\infty,\ f(x) \to -\infty$

Maximum number of turns: 3

x-intercepts: (0,0), (2,0), (−3,0) y-intercept: (0,0)

Verify that each of your x-intercepts are correct using the Factor/ Remainder theorems:

$-x \to x=0$

$-(0)(0-2)^2(0+3)$

$= 0(2)^2(3)$

$= 0(4)(3)$

$= 0$ ✓

$(x-2) \to x=2$

$-(2)(2-2)^2(2+3)$

$= -2(0)^2(5)$

$= -2(0)(5)$

$= 0$ ✓

$(x+3) \to x=-3$

$-(-3)(-3-2)^2(-3+3)$

$= 3(-5)^2(0)$

$= 3(25)(0)$

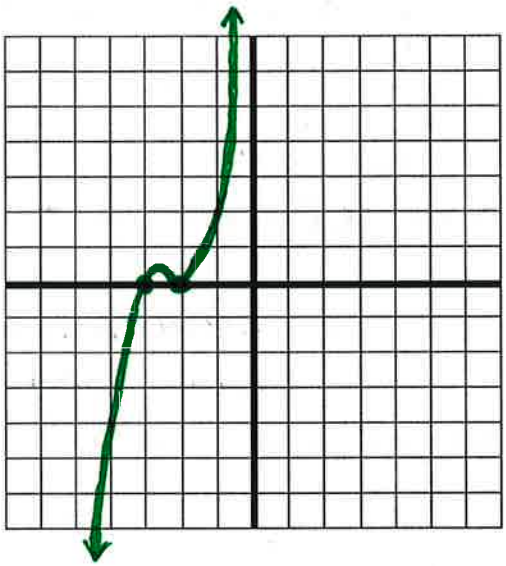
$= 0$ ✓

Create an XY table with all intercepts included

X	-4	-3	-2	-1	0	1	2	3	4	5
y	-144	0	32	18	0	-4	0	-18	-112	-360

12. Graph the polynomial below. *Hint: factor first.*

$$f(x) = x^3 + 7x^2 + 16x + 12$$



Classify *(by degree & term)*:

Degree: 3 Even / (Odd); LC: + + / -

End Behavior: $x \to \infty,\ f(x) \to \infty$

$x \to -\infty,\ f(x) \to -\infty$

Maximum number of turns: 2

Use Polynomial Division to FACTOR COMPLETELY if $x = -3$

	x^2	$4x$	4
x	x^3	$4x^2$	$4x$
3	$3x^2$	$12x$	12

$f(x) = (x+3)(x^2+4x+4)$

	x	2
x	x^2	$2x$
2	$2x$	4

Mult: $4x^2$

Add: $4x$

$f(x) = (x+3)(x+2)^2$

x-intercepts: (-3,0), (-2,0) y-intercept: (0,12)

Verify that each of your x-intercepts are correct using the Factor/ Remainder theorems:

$(x+3) \to x = -3$

$(-3)^3 + 7(-3)^2 + 16(-3) + 12$

$= -27 + 7(9) - 48 + 12$

$= -27 + 63 - 48 + 12$

$= 0$ ✓

$(x+2) \to x = -2$

$(-2)^3 + 7(-2)^2 + 16(-2) + 12$

$= -8 + 7(4) - 32 + 12$

$= -8 + 28 - 32 + 12$

$= 0$ ✓