

Unit 7 Test Study Guide

1. Perform the indicated operations. Be sure to indicate the excluded values and least common denominator when necessary.

$$A.) \frac{x^2-25}{x^2+2x-15} \cdot \frac{3x^2-9x}{x+3}$$

$$= \frac{(x-5)(x+5)}{(x+5)(x-3)} \cdot \frac{3x(x-3)}{x+3}$$

$$= \frac{3x(x-5)}{x+3} = \boxed{\frac{3x^2-15x}{x+3}}$$

exclude: $x = \pm 3, -5$

$$C.) \frac{x+3}{2x^3} - \frac{5}{8x^2} \cdot x$$

$$= \frac{4x+12}{8x^3} - \frac{5x}{8x^3} = \boxed{\frac{12-x}{8x^3}}$$

exclude: $x=0$

$$E.) \frac{x^2-6x+8}{x^2-8x+12}$$

$$= \frac{(x-4)(x-2)}{(x-6)(x-2)} = \boxed{\frac{x-4}{x-6}}$$

exclude: $x = 2, 6$

LCD: $(x-6)(x+2)$

$$B.) \frac{7x}{x^2-4x-12} + \frac{3}{x-6}$$

$$= \frac{7x}{(x-6)(x+2)} + \frac{3}{x-6} \cdot \frac{(x+2)}{(x+2)}$$

$$= \frac{7x+3x+6}{(x-6)(x+2)} = \boxed{\frac{10x+6}{(x-6)(x+2)}}$$

exclude: $x = -2, 6$

$$D.) \frac{5xy^1}{x^2y^8} \div \frac{10xy^2}{6x^4y^5}$$

$$= \frac{5xy}{x^2y^8} \cdot \frac{6x^4y^5}{10xy^2} = \frac{30x^5y^6}{10x^3y^{10}} = \boxed{\frac{3x^2}{y^4}}$$

exclude: $x=0, y=0$

$$F.) \frac{4x^2-16x}{x+8} \div \frac{x^2-x-12}{2x+16}$$

$$= \frac{4x(x-4)}{x+8} \cdot \frac{2(x+8)}{(x-4)(x+3)} = \boxed{\frac{8x}{x+3}}$$

exclude: $x = -8, -3, 4$

2. Solve the rational equations below. Determine any the excluded value(s).

A.) $\frac{-5}{x-7} = \frac{x}{x-3}$

$-5(x-3) = x(x-7)$

$-5x + 15 = x^2 - 7x$

$0 = x^2 - 2x - 15$

$0 = (x-5)(x+3)$

$x = -3, 5$

Exclude: $x = 3, 7$

B.) $\frac{x}{x-7} + \frac{3}{x+3} = \frac{x^2}{x^2-4x-21}$

$x(x+3) + 3(x-7) = x^2$

$x^2 + 3x + 3x - 21 = x^2$

$6x = 21$

$x = \frac{21}{6}$

Exclude: $x = -3, 7$

3. Given the equations or graphs of the following rational functions, determine the indicated key features.

A.) $f(x) = \frac{x+3}{2x+5}$

Horizontal Asymptote: $y = \frac{1}{2}$

Vertical Asymptote(s): $x = -\frac{5}{2}$

X-intercept(s): $(-3, 0)$

Domain: $(-\infty, -\frac{5}{2}) \cup (-\frac{5}{2}, \infty)$

Range: $(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

B.) $f(x) = \frac{x-5}{x^2+5x+4} = \frac{x-5}{(x+4)(x+1)}$

Horizontal Asymptote: $y = 0$

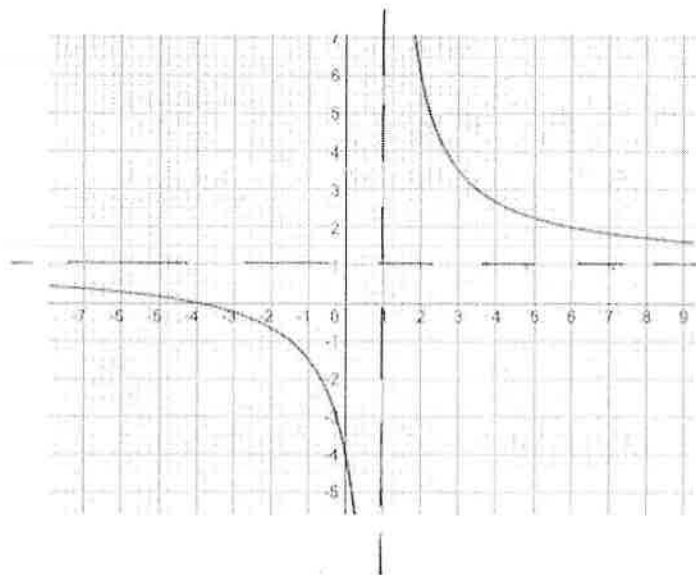
Vertical Asymptote(s): $x = -4$ and $x = -1$

X-intercept(s): $(5, 0)$

Domain: $(-\infty, -4) \cup (-4, -1) \cup (-1, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

C.)



Horizontal Asymptote: $y = 1$

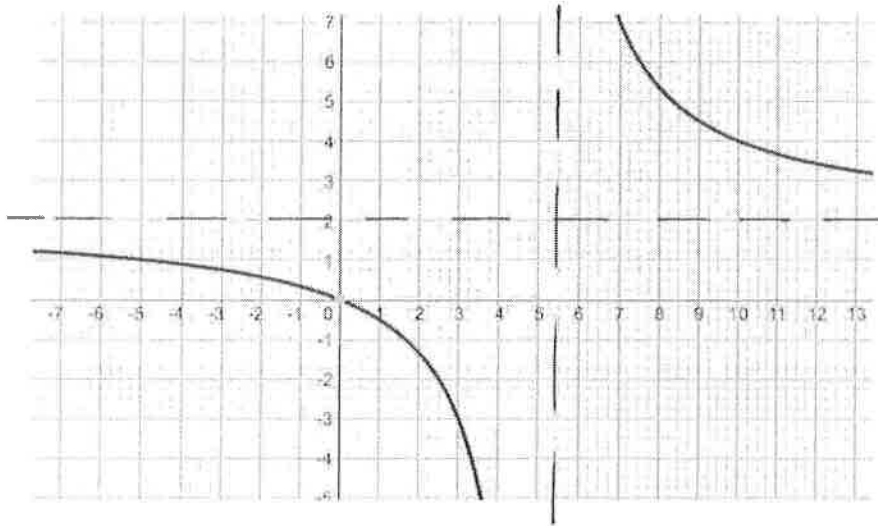
Vertical Asymptote(s): $x = 1$

X-intercept(s): $(-4, 0)$

Domain: $(-\infty, 1) \cup (1, \infty)$

Range: $(-\infty, 1) \cup (1, \infty)$

D.)



Horizontal Asymptote: $y=2$
 Vertical Asymptote(s): $x=5.5$
 X-intercept(s): $(0,0)$
 Domain: $(-\infty, 5.5)(5.5, \infty)$
 Range: $(-\infty, 2)(2, \infty)$

4. Find the inverse for each of the following

A.) $f(x) = \frac{3}{2x-5}$

$$x = \frac{3}{2y-5}$$

$$x(2y-5) = 3$$

$$2xy - 5x = 3$$

$$2xy = 5x + 3$$

$$y = \frac{5x+3}{2x}$$

$$f^{-1}(x) = \frac{5x+3}{2x}$$

C.) $g(x) = \frac{x-3}{4x}$

$$x = \frac{y-3}{4y}$$

$$4xy = y-3$$

$$4xy - y = -3$$

$$y(4x-1) = -3$$

$$y = \frac{-3}{4x-1}$$

$$g^{-1}(x) = \frac{-3}{4x-1}$$

B.) $h(x) = \frac{2x+7}{3x-1}$

$$x = \frac{2y+7}{3y-1}$$

$$x(3y-1) = 2y+7$$

$$3xy - x = 2y+7$$

$$3xy - 2y = x+7$$

$$y(3x-2) = x+7$$

$$y = \frac{x+7}{3x-2}$$

$$h^{-1}(x) = \frac{x+7}{3x-2}$$

5. Write the equation of a rational function given the following characteristics:

A.) A vertical asymptote at 4, a horizontal asymptote at $y = \frac{9}{12}$, an x-intercept at (7, 0).

$$f(x) = \frac{9(x-7)}{12(x-4)} = \frac{3(x-7)}{4(x-4)} = \frac{3x-21}{4x-16}$$

B.) Excluded values of $x = 3$ and $x = -1$, a vertical asymptote at $x = 3$, an x intercept at (6, 0) and a horizontal asymptote at $y = 1$.

$$f(x) = \frac{(x-6)(x+1)}{(x-3)(x+1)}$$